

TRANSIT NETWORK DESIGN FOR AREAS WITH LOW
AND/OR HETEROGENEOUS DEMAND

BY

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DISSERTATION

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Abstract

Low density and spatial heterogeneity in transit demand impose considerable challenges to both transit riders and service agencies. For example, lower demand forces transit agencies to provide sparser and less accessible service so as to stay economical, which however further deteriorates passenger experience and deters patronage. Spatially heterogeneous demand as well as city street characteristics (e.g., network layout) calls for variation in transit service, which often leads to higher system costs and undesirable passenger experience (e.g., transfers) as well.

This dissertation proposes a series of transit network design methods that can be used to improve transit service under these circumstances. It first presents an alternative flexible-route transit system for low demand areas, in which each bus is allowed to travel across a predetermined area to serve passengers, while these bus service areas collectively form a hybrid “grand” structure that resembles hub-and-spoke and grid networks. We analyze the agency and user cost components of this proposed system in idealized square cities and seek the optimum network layout, service area of each bus, and bus headway to minimize the total system cost. We compare the performance of the proposed transit system with that of other conventional systems (e.g., fixed-route transit network and taxi service), and show which system is advantageous under different passenger demand levels. It is found that under low-to-moderate demand levels, the proposed flexible-route system has the lowest overall system cost.

In the second part of this dissertation, a methodological framework is developed so that continuum approximation (CA) techniques can be used to design bus networks for cities where travel demand varies gradually over space. The bus-route configurations consist of (i) a main,

city-wide grid with relatively large physical spacings between its parallel routes and the stops along those routes; together with (ii) one or more local grids with more closely-spaced routes and stops that serve neighborhoods of higher-demand densities. The so-called power-of-two concept is borrowed from the field of inventory control, and is enforced so that the local grids can be embedded seamlessly within the main one. Numerical experiments illustrate the value of the resulting heterogeneous route configurations, which have the potential to reduce the costs to both the bus users and the operating agency, as compared against the costs of the optimal homogeneous bus-route grids. Differences of about 5-10% are observed for a set of numerical examples that cover a wide range of demand distribution patterns. Most of the savings are due to the diminished access costs that users incur when high-demand neighborhoods are served by local grids with closely-spaced routes and stops.

The same CA methodology is used to design a simplified grid network system with variable bus spacings that can address spatially heterogeneous demand. The continuum approximation enables us to estimate the cost components of the system locally and design the system layout accordingly. The simplified grid system varies the bus line spacings, which makes the network more responsive to the varying demand. It saves user costs in relatively high demand areas and saves agency costs in lower demand areas. Numerical results show that the design can improve the total cost of the system by between 4% and 6% as compared with traditional grid designs under the chosen demand distribution patterns.

We further extend the network design framework from grid city street networks to a radial one, where buses can travel either radially (from center to outer part and vice versa) or circularly (clockwise and counterclockwise along rings). Using the CA method, we analyze user and

agency costs and propose a design framework that minimizes the total system cost. The optimum design provides the bus spacings of the radial and circular lines and the bus headway. Numerical results show that the proposed method can be useful to design an efficient system when the city streets form a ring-radial network.

The last part of this dissertation is devoted to real-world applications of the proposed design methods. We apply the flexible-route transit network to satisfying evening/night demand in the campus area of Urbana-Champaign, which is currently covered by a manually dispatched “Safe-Ride” system. The new transit system is shown to outperform the current system and achieve several improvements. We also design a new transit network for the City of Weihai in China, where the network includes both grid and polar network components to best serve the city’s geographical layout. Numerical experiments also give insights on the influence of key decision variables such as bus line spacing, stop spacing, and headway under different design schemes.

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Chapter 1

Introduction

1.1. Motivation

Transit network design is a critical problem for all the public agencies. The agencies try to provide bus coverage for the entire service region to serve passengers' needs. The municipality agencies need to plan their public transit system by designing the master plan of the bus route as well as the schedule of the buses. One of the challenges associated with transit network design is the variation of demand over time and space. The demand changes during different hours of a day significantly as well as different days of a week (weekdays and weekends). This variation over time requires designing different transit systems for different periods (e.g., fixed-route and flexible-route for higher and lower demand rate respectively). Usually the transit agency tries to keep their buses running as long as there is enough demand to serve.

Mainly the transit agency designs a fixed-route system based on high demand hours (e.g., from early morning till the evening). During other time, the agency may discontinue some or all of their buses or reduce their frequency. The bus system can be designed as fixed-route for high demand and flexible-route for low demand (when fixed-route is not economic). In this regard, one of the interesting questions is to find the threshold for fixed versus flexible route. Unlike the fixed-route transit network, designing flexible-route transit system that can potentially be used for low demand is one topic that has not been well explored in the literature before.

The first step of transit network design is choosing the proper layout for the transit network. Based on the shape and structure of the service region, different possibilities can be considered as a transit network layout. The grid network is the simplest one in which the buses travel in N-S and E-W directions. Hub-and-spoke is another popular network layout which is common in historical cities. In addition, other transit network layout designs (e.g., radial or multi hub-and-spoke) could be considered. Restricting the design to only one general layout may not be optimum. There are innovative ideas that combine the different layouts to get the advantages of each of them (e.g., combination of grid and hub-and-spoke into a hybrid network; see Daganzo 2010).

In addition to demand variation over time, the demand may also vary over the space. For simplicity, some existing literature assumes uniform demand over the space, but that is not realistic in many cases. To design a transit system while considering heterogeneous demand is another motivation of this dissertation.

Transit network design is in nature a complicated problem and there are many factors that should be taken into account. For example the inconvenience caused by the number of transfers for passengers should be considered in addition of waiting time spent at transfer points. In this dissertation we aim to address all these factors in comprehensive and realistic models, while at the same time, we make reasonable assumptions to maintain simplicity.

1.2. Objectives

The main objective of this Ph.D. dissertation is to design a transit network system that minimizes the total system costs including both transit agency and passenger costs. We investigate the transit network design problem for both flexible-route and fixed-route transit systems. In addition, we consider different service region layouts, shapes, and city streets, as well as heterogeneous demand distribution over space.

To be more specific we first study the flexible-route transit network design that can be economic for lower demand areas. We apply flexible-route where buses pick up and drop off passengers, while we consider a hybrid of grid and hub-and-spoke networks as the overall layout of the system. We analyze all the cost components in order to design the optimum transit system that gives us the transit system specifications including network layout.

We also devote part of this study to analyze the fixed-route transit network. We are looking into the demand variation over the space, and based on that we propose a grid transit network design that is capable of addressing this demand variation either using variable bus spacings or adding local lines. In the variable bus spacings part, we design narrow bus spacings in relatively dense areas and wide spacing in relatively low demand areas in order to reduce the total cost (as compared with uniform spacings) over the entire service region. In local line design, the main concept is to add extra bus lines in higher demand areas and synchronize them with existing bus lines.

In the last technical part of this dissertation we study polar transit systems in mono-centric cities with ring-radial city streets. We attempt to extend our analysis method to design a transit network for circular shape and ring-radial street network cities. We analyze the transit system

cost components, based on which we design the optimum transit network. We also apply the design procedure to hypothetical cases.

For all systems, we apply the proposed models to numerical examples (either real case studies or hypothetical examples) to illustrate how the proposed transit system design works. We further make comparisons among the proposed systems and the transitional ones to show the improvements in user, agency and total system costs.

1.3. Contribution statement

This Ph.D. research studies flexible-route and fixed-route transit network design considering heterogeneous demand distribution over space. Part of this study is devoted to developing a structured flexible-route transit network design for low demand levels and the other part investigates the fixed-route transit system. In the flexible-route transit network design part, we propose the design of flexible-route transit system using the hybrid of grid and hub-and-spoke as the network layout. We obtain closed-form formula for all cost components including transit agency and passengers. Knowing all cost components enables us to design the optimum transit network. We apply the proposed methodology for hypothetical cases and we plan to extend it to real-world cases.

The other part of this study dedicated to designing a transit system with local lines in a grid network context. We propose this idea to address the variable demand by adding extra bus lines to the main grid bus lines where the demand is high. For synchronization and spatial matching of the local and main bus lines, we propose using the power-of-two concept adapted from inventory

control. In addition of the idea of local bus lines, we propose a simplified grid transit network with variable bus spacings. Similarly, we analyze the system cost components and design an efficient transit system based on optimum system cost. We further extend our analysis method and the concept of bus lines with variable spacings to polar transit network. We present methodological framework for designing a polar transit system that buses travel radially (from center to outer part and vice versa) and circularly (clockwise and counterclockwise).

In all the fixed-route cost component analysis, we apply CA method to analyze the system cost components including user and agency costs. This method enables us to analyze the cost components locally and obtain closed-form formula. We further develop heterogeneous designs over space based on minimum cost. Using this powerful tool, we design transit network that can minimizes the total system costs.

In summary, in practical point of view we propose new approaches in transit network design that can be applied to many cities. In academic point of view we propose the application of CA method in transit network design which provides us closed-form formula for all cost components locally and for whole the system. Moreover, we apply the both flexible- and fixed-route to real-world cases.

1.4. Outline

This dissertation is organized as follows. Chapter 2 reviews the related literature and discusses more about the traditional method versus the CA method. Chapter 3 proposes a structured flexible-route transit system for low demand areas. Chapter 4 investigates a transit system design

for demand varying functions over the space including transit system design with local lines. Chapter 5 proposes a simplified grid network system with variable bus spacings. Chapter 6 studies polar transit network design for ring-radial cities. Chapter 7 and 8 design flexible-route transit system for Urbana-Champaign and fixed-route transit system for Weihai, respectively. Finally, Chapter 9 provides concluding discussions and future research directions.

Chapter 2

Literature Review

This chapter first reviews the transit system design literature, including some key methodologies. Then we briefly introduce the CA method which we apply to transit system design models in Chapters 4-6.

2.1. Transit network design

Transit network design is one specific application of transportation network design problem. Network design problems have been extensively studied over the past few decades. Comprehensive reviews can be found in e.g, Friesz (1985) and Yang and Bell (1998). The general transportation network design model can be formulated as follows (Magnanti, 1984):

$$\min \phi(f, y) \tag{1.a}$$

subject to

$$\sum_{j \in N} f_{i,j}^k - \sum_{l \in N} f_{l,i}^k = \begin{cases} R_k & \text{if } i = O(k) \\ -R_k & \text{if } i = D(k), \forall k \in K, \\ 0 & \text{otherwise} \end{cases} \tag{1.b}$$

$$f_{i,j} = \sum_{k \in K} f_{i,j}^k \leq K_{i,j} y_{i,j}, \forall (i, j) \in A, \tag{1.c}$$

$$(f, y) \in S, \tag{1.d}$$

$$f_{i,j}^k \geq 0, \quad y_{i,j} = \{0,1\}, \quad \forall (i,j) \in A, \quad k \in K, \quad (1.e)$$

where N and A are the sets of nodes and arcs, respectively. Furthermore, set K denotes the commodities and R_k presents the required flow at origin $O(k)$ and destination $D(k)$. We let $y_{i,j}$ be the binary decision variable which is 1 if arc (i,j) is chosen as a part of network design, and 0 otherwise. The continuous decision variable $f_{i,j}^k$ denotes the flow of commodity k on arc (i,j) and parameter $K_{i,j}$ shows the capacity of arc (i,j) . The objective function (1.a) is general and can be any function. For instance, it can be in the form of a linear function, as follows:

$$\phi(f, y) = \sum_{k \in K} \sum_{(i,j) \in A} c_{i,j}^k f_{i,j}^k + \sum_{(i,j) \in A} F_{i,j} y_{i,j}, \quad (2)$$

where $c_{i,j}^k$ is the cost of commodity k and F_{ij} is the fixed cost of constructing arc (i,j) . The set S in constraint (1.d) can enforce any constraint based on the network design.

On the other hand, transit network design problem has been widely studied in the literature. Several review papers have summarized the transit route network design problem and investigated different aspects of this domain. For example, the most recent literature by Kepaptsoglou and Karlaftis (2009) presented a review research paper on Transit Route Network Design Problem (TRNDP) with the main focus on design objectives, operation environment parameters, and solution approaches. Guihaire and Hao (2008) classified and reviewed the design and scheduling of the network including network design, frequency setting, timetable development, bus and driver scheduling. Desaulniers and Hickman (2007) reviewed public transit design including network design, timetabling, vehicle scheduling, and crew scheduling in operation research point of view. They also summarized a variety of optimization problems that

are related to public transit. Fan and Machemehl (2004) reviewed different transit route network design problems and proposed a model and solution algorithm. Zhao and Gan (2003) presented a review report on the previous studies and developed a model and heuristic solution technique to minimize the number of passengers' transfers. They also applied their model to some numerical test cases that showed their solution could reduce the average number of transfers. We can also refer to Ceder and Wilson (1986) for a review of different aspects of bus network design models and solution techniques. Chua (1984) reviewed and compared different approaches of urban bus network planning and conducted a survey on the British urban bus operators.

More specifically, the topology of bus networks has been studied as early as the 1960s (e.g., Ceder and Wilson, 1968). The research for a cost-minimizing urban transit system structure (e.g., route positions and headways) explored possible grid networks (Holroyd, 1965), radial systems (Byrne, 1975), and hub-and-spoke systems (Newell, 1979). Wirasinghe et al. (1977) found the optimal network parameters to minimize transit operating cost and passenger travel time in coordinated rail and bus transit systems. Mandl (1980) proposed a method to compute the passenger's transportation cost and apply it for improving the transit network. Ceder and Israeli (1998) presented a nonlinear mixed-integer program in which the objective function includes both passenger and operator interests. Fan and Machemehl (2006) proposed a multi-objective non-linear mixed integer model and used heuristic methods to solve it. Their objective function is the minimization of both user and agency cost considering unsatisfied demand. To solve that problem they generated an initial route and used genetic algorithm to improve it. The formulation of the transit route construction by Ceder (2007) is presented as follows.

$$\min \sum_{r \in R} c_r x_r + \sum_{tr \in TR} c_{tr} \pi_{r:tr} x_r \quad (3.a)$$

$$\sum_{r \in R} a_{i,j}^r x_r + \sum_{f \in F} a_{i,j}^{tr} \pi_{r:tr} x_r \geq 1, \quad \forall i, j \in N, \quad (3.b)$$

$$c_r = \sum_{i,j \in N_r} (t_{i,j}^r - t_{i,j}^{sp}), \quad \forall r \in R, \quad (3.c)$$

$$c_{tr} = \sum_{i,j \in N_{tr}} (t_{i,j}^{tr} - t_{i,j}^{sp}), \quad \forall tr \in TR, \quad (3.d)$$

$$\sum_{r \in R} a_{i,j}^r + \sum_{f \in F} a_{i,j}^{tr} \geq 1, \quad \forall i, j \in N, \quad (3.e)$$

$$x_r = \{0,1\}, \quad \forall r \in R, \quad (3.f)$$

$$a_{i,j}^r = \{0,1\}, \quad \forall r \in R, \quad (3.g)$$

$$a_{i,j}^{tr} = \{0,1\}, \quad \forall tr \in TR, \quad (3.h)$$

The objective function introduces the set-covering problem, where it creates a minimum set of routes and their related transfers. Decision variable x_r is 1 if route $r \in R$ is in the solution, and 0 otherwise. $a_{i,j}^r$ ($a_{i,j}^{tr}$) is 1 if the demand (i,j) can be handled directly by route $r \in R$ (by transfer path $tr \in TR$) and, 0 otherwise. Constraints (3.c) and (3.d) define c_r and c_{tr} as the cost of direct route and transfer path, respectively. Note that $t_{i,j}^r$, $t_{i,j}^{tr}$, and $t_{i,j}^{sp}$ show the average travel time between i and j on route r , on transfer path tr and on its shortest path, respectively. The sets R , TR , S , N_r , N_{tr} , N_{sp} present the transit routes, transfer paths, shortest path, nodes located on route r , nodes located on transfer path tr , and nodes located on shortest path sp , respectively.

The inputs of transit network design model tend to be voluminous, primarily because the inputs entail a good many site-specific details about the system and its operating environment (e.g., Shrivastava and O'Mahony (2006)). Considering many factors as input parameters, solutions (i.e., determinations of the designs that minimize generalized costs) require heuristics and these may come with significant computational costs (e.g., Bagloee and Ceder (2011)).

Furthermore, there are a number of other studies that focused on specific aspects of transit network design, such as mass transit design (Barnett, 2010), bus priority at signalized intersections (Balke et al., 2000), and bus lane priority (Eichler and Daganzo, 2006).

Besides the traditional transit network design, there are some innovative research on system layout or analysis. Models of this type have been used in the design of so-called hybrid transit networks in which routes in the city center are laid-out in a grid structure so that stops enjoy four-directional (N-S and E-W) transit service, while the city's periphery is covered by a hub-and-spoke structure (Newell, 1979; Wirasinghe, 1977) with two-directional service (Daganzo, 2010a). Compared to the network that features only a grid structure (Holroyd, 1965) or a hub-and-spoke structure (Newell, 1979; Wirasinghe et al., 1977), a hybrid design can often provide service at a lower generalized cost. Subsequent works have shown how an idealized hybrid design can be tailored to suit a real city's topography (Estrada et al, 2011). Most of these models treat travel demand as a target that is exogenous to the details of the network design. This is not as serious a concern as some readers may think because generalized cost is rather insensitive to demands that vary about the true value (Daganzo, 2010b; 2012).

Moreover, there is some literature that studied the polar transit system design in ring-radial cities. Byrne (1975) developed a transit network model that minimized the user and agency cost.

Morlok and Viton (1984) studied radial urban transit service that can maximize the profit. Vaughan (1986) analyzed the polar network to minimize the total passenger travel time subject to fleet constraints and presented a simple numerical example. There are also some other literature that discuss the transit network design in polar system as a part of their research (e.g., Holroyd (1965) and Black (1978)).

2.2. Continuum Approximation (CA) models

The Continuum Approximation (CA) method for solving transportation problems was first proposed by Newell (1971) for finding optimum headway. He also applied CA method for other transportation fields such as vehicle scheduling or facility location problems (1973). Figure 2.1 shows the cumulative number of arriving and departing passengers for a scheduled transportation system. In this figure, $A(t)$ is the cumulative number of passengers to arrive by time (t), the step function $D(t)$ is the cumulative number of passengers, and $D^*(t)$ is the smooth approximation of the $D(t)$ that can be used in CA method. In this context, Daganzo and Newell (1986), using CA method, determined the number of transshipment points and vehicles routing in a network. In the past three decades CA was applied to different aspects of transportation problems. Langevin et al. (1996) and Daganzo (2005) provide an overview of the application of CA method to designing freight distribution systems.

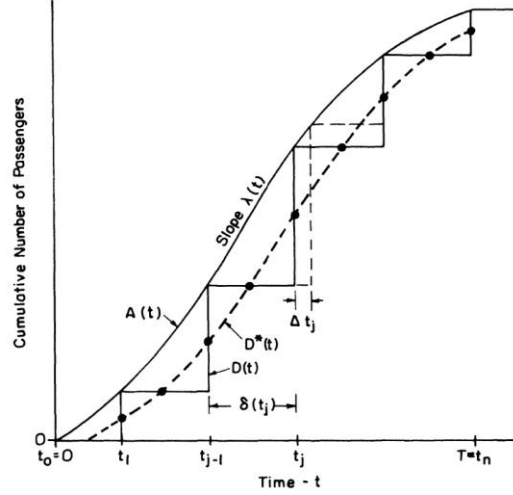


Figure 2.1 Cumulative number vs. time for a transportation system (Newell, 1973).

Daganzo (2005) used the CA method to solve lot sizing problems with variable demand. The CA method can efficiently solve this problem to near-optimal. Figure 2.2.a illustrates the problem which is the cumulative number of items dispatch over time. Curves $D(t)$ and $R(t)$ are the rates of items consumed and received, respectively. Figure 2.2.b illustrates the construction method for cumulative number of items shipped versus time, and by using CA method Figure 2.2.c shows the discretization method for obtaining a set of dispatching times, $H_s(t)$ from continuous dispatching time function, $H(t)$. $H(t)$ can be obtain from the following equation.

$$H(t) = \left[\frac{2c_f}{c_i D'(t)} \right]^{1/2}, \quad (2)$$

where c_i and c_f are inventory cost and fixed cost per vehicle dispatch.

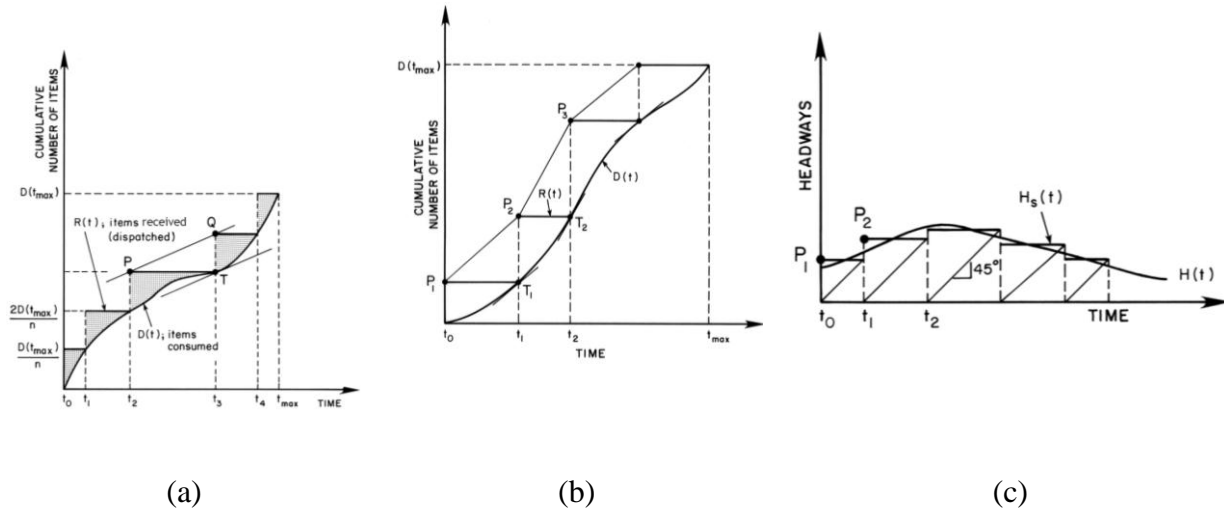


Figure 2.2 Lot size problem with variable demand (Daganzo, 2005).

CA algorithm has been used to solve logistics problems including facility location problems as well as network flow problems. Wong (1998) applied CA method for solving network flow of traffic assignment with variable demand. Ouyang and Daganzo (2006) used CA method to solve two dimensional location problem. Later, Li and Ouyang (2010) applied CA method for reliable facility location problem.

In this dissertation, we plan to develop CA modeling framework as a part of solution approaches to transit network design. The general idea of CA method for transit network design is to decompose and calculate the cost components at separate neighborhoods. We assume smooth demand function and based on that we calculate the cost components. Then we can apply optimization techniques to minimize the total system-wide cost over the service region and finally use discretization method to design the transit network.

Chapter 3

A Structured Flexible Transit System for Low Demand Areas¹

3.1. Introduction

Traditionally, transit systems are designed to contain fixed bus routes, and passengers move to predetermined stations to gain access to the system. Very recently, Daganzo (2010) proposed an innovative transit network design framework that determines the adequate structure of the network as well as the optimal headway for a range of transit modes. It was shown that such a hybrid network structure (see Figure 3.1) nicely inherits the advantages of both a hub-and-spoke structure (e.g., low infrastructure investment) and a grid structure (e.g., low travel time).

¹ This Chapter has been adopted from “Nourbakhsh, S.M., Ouyang, Y., 2012. A structured flexible transit system for low demand areas. *Transportation Research Part B* 46 (1), 204–216.”.

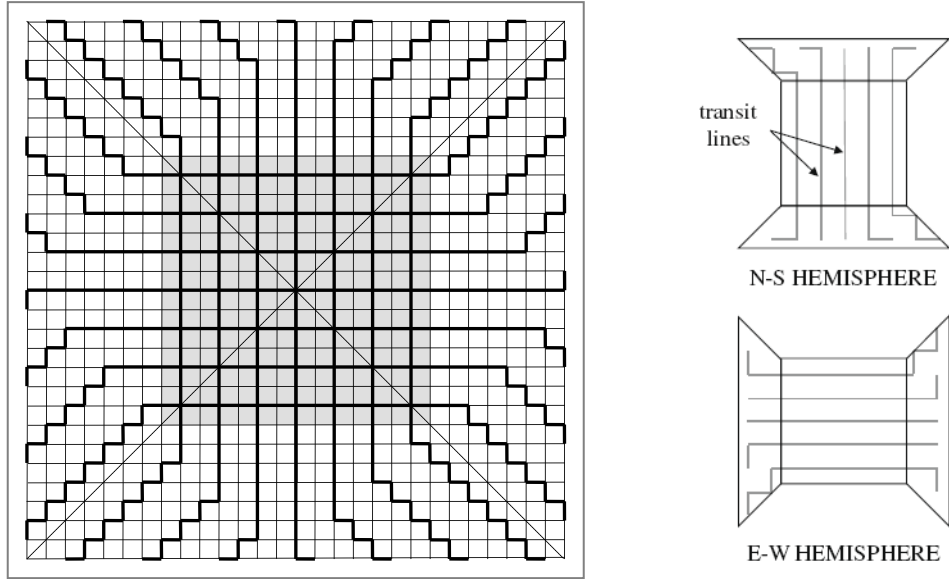


Figure 3.1 A hybrid network structure (Adapted from Daganzo [2010]).

Fixed routes in a transit system provide clarity and regularity to the transit service, and it is known to work very well for densely populated cities (i.e., with high passenger demand) in general. However, in low demand regions (e.g., sprawled suburban areas), the optimal spacing between bus routes often tends to be relatively large so as to reduce the total system cost. As such, the transit system exposes passengers to the open (sometime adverse) environment for a long time while they walk to and from the bus stops. In such cases, adding flexibility to transit route and schedule seems desirable. For example, Quadrifoglio et al. (2006) demonstrated the potential advantages of allowing transit vehicles to travel within a low-demand geographical region to pick up and drop off passengers. Properties of such operations (e.g., bounds on the maximum longitudinal velocity) are obtained, often via simulations, and incorporated into the overall network design via mixed-integer programming models (e.g., Quadrifoglio et al., 2008).

For feeder buses, the suitable demand density for the flexible-route operation is also studied (Quadrifoglio and Li, 2009, among others).

This chapter aims to integrate these interesting ideas (e.g., hybrid network structure, flexible route) into the design of a new structured “flexible-route transit system.” Individual buses operate without fixed routes or predetermined stops, but rather they can travel around within their own service regions to pick up or drop off passengers. At the macroscopic level, however, the buses (or their service regions) collectively form a suitable network structure to provide reliable spatial and temporal service coverage to the entire demand area. In this chapter, instead of relying on complex mathematical programs to determine the optimal network structure numerically, we express the system’s operating performance into analytical functions of a few key design variables, and solve for the optimal design as a simple constrained nonlinear optimization problem. These analytical functions also cast important insights into the impacts of these design variables on the overall system performance. Numerical examples and comparison with comparable alternative systems (e.g., the fixed-route transit system and taxi service) show that the proposed flexible-route transit system is advantageous under a range of low-to-moderate demand levels. This is encouraging because the proposed transit system can be used in a number of real-world applications. One possibility, for example, is to let the transit system switch among different operating modes according to the demand level at specific hours (e.g., at night or during weekend). In addition, the flexible network can be used for design of “safe ride” or “dial-a-ride” systems in which passengers call and wait for pick-ups.

The exposition of this chapter is as follows. Section 3.2 introduces the notation, concept and formulation of the flexible-route transit system. Section 3.3 examines other transit system structures (including the fixed-route transit system and a taxi system) that are comparable to the

proposed one. In Section 3.4, the proposed system is numerically compared with the other systems at different demand levels. Finally, Section 3.5 provides conclusions.

3.2. Methodology

3.2.1. Notation and definition

Similar to Daganzo (2010), we consider a square service region of side D (km) that generates λ passenger trips per hour per unit area. The trip origins and destinations are uniform and independently distributed in the region according to a homogeneous spatial Poisson process. The local streets in this service region align along a grid network with constant spacing s .

We consider designing a new flexible transit system to provide service to the passengers when λ is relatively small. Unlike the traditional transit system where buses travel along a fixed route and make stops at predetermined stations, in the flexible transit system buses pick up passengers at their origins or drop them off at their destinations. Each bus now serves the passengers in a narrow elongated area, which we call it a “bus tube,” as shown in Figure 3.2(a). The bus makes lateral movements while sweeping longitudinally back and forth through the tube. The exact bus trajectory obviously depends on the realization of passenger locations, but fixed transfer points are planned along each bus tube, and we assume buses always stop at those points.

Figure 3.2(b) shows the grand overview of the structured flexible-route transit system, including the layout of tubes and the location of transfer points. Based on the intended service level, the whole demand area can be divided into the central square and the peripheral quadrants. The transit system includes N-S and E-W hemispheres each containing N equal transit tubes

(with variable width), providing double coverage in the central square and single coverage in the peripheral part². Obviously, there are N transfer points in each tube (in the inner square) where passengers may transfer to other buses. The central square has a side length of d , and we use the ratio $\alpha = \frac{d}{D}$ to indicate the relative size of the inner square. The maximum width of the tubes (at the boundary of the entire area) is $w = \frac{D}{N}$, and the width of the tube inside the central square equals to αw . We further assume that each bus travels in its tube with headway H .

Our design problem is to find the optimal combination of decision variables, $\alpha \in [0,1]$, $H \geq 0$, and $N \in \{1,2,\dots\}$, that minimize the total system costs for the proposed transit system. We further assume that the bus cruising speed is v , the time needed to make one stop (i.e., the delay for acceleration and deceleration) is τ_1 , the time needed to pick up or drop off a passenger is τ_2 . Since we assume one stop per passenger, the time needed per passenger stop is τ , where $\tau = \tau_1 + \tau_2$. As a result, the average bus travel speed (along its flexible trajectory) reduces to $v_c \leq v$. Let C be the capacity of the buses, and δ be the factor that captures the discomfort associated with transfers. Passenger can walk at speed v_w .

² Even though the passenger origins and destinations are assumed to be homogeneous over space and time, it is reasonable to design a higher density of transit network infrastructures near the central part of the city (where most of the traffic is expected to traverse). In the context of many-to-many freight logistics systems, Campbell (1990) showed that the optimal transshipment terminals shall be evenly spaced and yet clustered at the center of the service region.

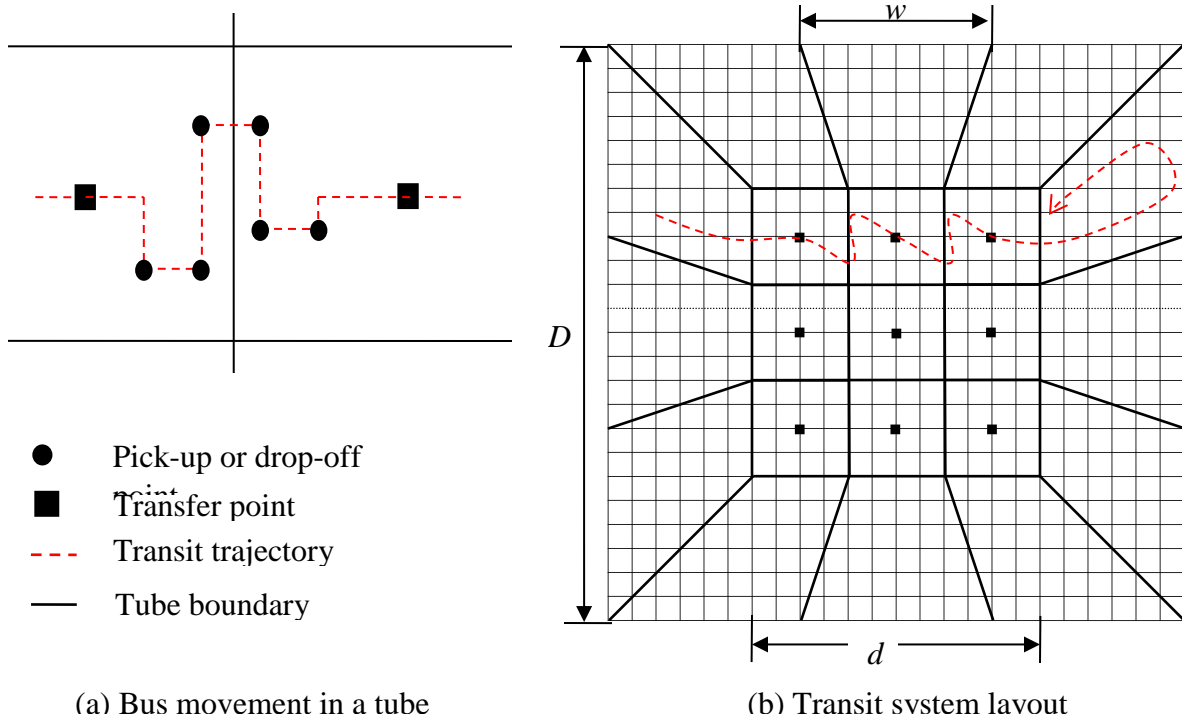


Figure 3.2. General scheme of the structured flexible transit system.

3.2.2. Formulation

The total system cost mainly consists of two parts: user costs and agency costs. In the following subsections we present formulas for these cost components. Detailed derivations of these formulas can be found in Appendix A.

3.2.2.1 Agency costs

The agency costs include the expected total vehicle distance traveled per hour of operation, Q , and the expected total fleet size in operation, M . Unlike the fixed-route transit system, the proposed flexible-route system has no specific bus routes or specific locations for picking up or

dropping off passengers, and thus the need for capital infrastructure investments (e.g., building bus stations) is minimal.³ Appendix A shows that the following formula holds for Q :

$$Q = \frac{2N}{H} \left[D \sum_{i=2}^{\infty} (i-1) [\alpha P_c\{i\} + (1-\alpha)P_p\{i\}] + 2D + \frac{2\lambda HD^3 \alpha^3}{3N^2} + \frac{2\lambda HD^2 (1-\alpha^2) l_p}{N} \right], \quad (1)$$

where

$$P_c\{i\} = \left(\frac{\alpha D s \lambda H}{N} \right)^i e^{-\frac{\alpha D s \lambda H}{N}} / i!, \quad (2)$$

$$P_p\{i\} = \left(\frac{(1+\alpha) D s \lambda H}{N} \right)^i e^{-\frac{(1+\alpha) D s \lambda H}{N}} / i!, \text{ and} \quad (3)$$

$$l_p = \begin{cases} \frac{(1+\alpha)D}{6N} + \frac{2N^3}{(1+\alpha)^3 D^3 \lambda^2 H^2} - \frac{4N^5}{3D^5 (1+\alpha)^5 \lambda^3 H^3}, & \text{if } D^2 (1+\alpha)^2 \lambda H \geq 2N^2 \\ \frac{N}{D(1+\alpha)\lambda H}, & \text{otherwise} \end{cases}. \quad (4)$$

The bus fleet size, M , is simply given by

$$M = Q / v_c, \quad (5)$$

where the bus average travel speed v_c is given by

$$\frac{1}{v_c} = \frac{1}{v} + \frac{2\tau\lambda D^2 H (1+\alpha^2)/N}{QH/2N}. \quad (6)$$

Interested readers are referred to Appendix B to see qualitatively how decision variables α , N , and H influence agency cost components Q and M .

3.2.2.2. User costs

³ There may be need for user interfaces (e.g., Internet or phone service systems) and driver communication devices (e.g., radio). But their costs are normally negligible compared with traditional roadway infrastructure investments.

The user costs are associated with (i) the total time for passengers to travel from their origins to their destinations, and (ii) other comfort-related factors such as the number of transfers. In the flexible transit system, the total travel time includes the passenger's waiting time at the origin and possibly at the transfer point(s), and the in-vehicle riding time. Because the passengers are picked up from their exact origins and dropped off at their exact destinations, there is no walking time.

Similar to Daganzo (2010), we assume that the passengers prefer fast and efficient travel. They always choose the travel plan with the least number of transfers, and if transfer is necessary, they transfer at the first opportunity. If there are choices with equal user costs (e.g., regarding the initial direction of travel), they break ties arbitrarily. We further assume that headway control strategies are implemented to stabilize bus schedules such that headway variations at the check points are negligible.⁴ Under these assumptions, it is shown in Appendix A that the expected waiting time, w , the expected number of transfers, e_T , the expected travel distance, E , the expected in-vehicle travel time, T , for a generic passenger, and the maximum expected vehicle occupancy, O , are given by the following formulas:

$$w = \frac{H}{2} \left(\frac{N-1}{N} \left(1 - \frac{\alpha^4}{N} \right) + \frac{(1-\alpha^2)^2}{2} + 1 \right), \quad (7)$$

$$e_T = \frac{N-1}{N} \left(1 - \frac{\alpha^4}{N} \right) + \frac{(1-\alpha^2)^2}{2}, \quad (8)$$

$$E \approx (4\phi(\alpha) + 5\varphi(\alpha))D + 2D \sum_{i=2}^{\infty} (i-1) [\phi(\alpha)P_c\{i\} + \varphi(\alpha)P_p\{i\}] + \frac{2\lambda HD^2}{N} \left(\frac{D\alpha^2}{3N} \phi(\alpha) + (1+\alpha)l_p \varphi(\alpha) \right), \quad (9)$$

⁴ Examples of such bus headway control strategies include adaptive holding (Daganzo, 2009) and adaptive cruise speed control (Daganzo and Palioskowski, 2011). Without headway control, the expected passenger waiting time at the transfer points may be larger than half of the headway due to length time bias (Daganzo, 1997).

$$T = \frac{E}{v_c} \quad (10)$$

$$O = \frac{\lambda H D^2}{N} \max \left\{ \frac{1 - \alpha^2}{2\alpha}; \frac{3 + 2\alpha^2 - 3\alpha^4}{8\alpha} + \frac{D}{w} \frac{(1 - \alpha^2)^2}{32} \right\}, \quad (11)$$

where $\phi(\alpha) = \frac{1}{12}(11\alpha - \alpha^3 - \alpha^5)$ and $\varphi(\alpha) = \frac{1}{18}(2 - 3\alpha + \alpha^3)$.

The maximum expected occupancy is not directly related to user costs. However, it could be useful to the transit agency such that buses with proper capacity (e.g. a van or a mini-bus) can be used.

3.2.2.3. Design

Appendix B shows that the user costs generally decrease with α , N and increase with H , but the agency costs generally increase with α , N and decrease with H . The optimization problem for the proposed system is to find the best decision variables α (central square size), N (the number of tubes in each direction), and H (bus headway) that balance the trade-offs between the agency and user costs;

We convert the agency costs into travel time equivalents. Suppose $\$Q$ is the agency operation cost per vehicle-distance, $\$M$ is the agency cost per vehicle hour, and μ is the average monetary value of one passenger-hour, then $\pi_Q = \frac{\$Q}{\lambda D^2 \mu}$ and $\pi_M = \frac{\$M}{\lambda D^2 \mu}$ convert the corresponding agency costs into the travel time equivalent per passenger (Daganzo, 2010). We let δ measure transfer discomfort, and then $\frac{\delta}{v_w}$ converts the expected transfer number e_T into passenger riding time. The optimization problem becomes the following:

$$\text{Min } z = \pi_Q Q + \pi_M M + W + T + \frac{\delta}{v_w} e_T \quad (12)$$

$$\text{s.t., } \alpha \in [\frac{1}{N}, 1], H \geq 0, N \in \{1, 2, \dots, \lfloor D/s \rfloor\}.$$

Note that we enforce $\alpha \geq \frac{1}{N}$ and $N \leq \lfloor D/s \rfloor$ (where $\lfloor \cdot \rfloor$ is the floor operation) because the central square should contain at least one transfer point, and each tube should contain at least one local street in the longitudinal direction.

To find the optimal value of the objective function, z , we may allow N to take a continuous value and apply numerical nonlinear optimization method (such as the steepest decent method). Numerical approximation is used to estimate the gradient of the objective function.

3.3. Other transportation system structures

In this subsection, we compare the performance of other service providing systems that are comparable to the proposed flexible-route transit system.

3.3.1. Fixed route transit network

In the fixed-route transit system, the total system costs include those related to infrastructure investment, total vehicle distance, bus fleet size, passenger walking time to and from the bus stations, passenger waiting time, and transfer discomfort. To be consistent with our flexible system, and to be conservative (i.e., favoring the fixed-route system), we disregard any possible infrastructure investment in the fixed-route system. The optimization objective can be expressed

in similar notations (while replacing each bus tube by a fixed bus route) as follows (Daganzo, 2010):

$$\text{Min } z = \pi_Q Q + \pi_M M + A + W + T + \frac{\delta}{v_w} e_T, \quad (13)$$

$$\text{where } Q = \frac{2D^2}{wH}(3\alpha - \alpha^2), \quad M = \frac{Q}{v_c}, \quad W = H \left(\frac{2 + \alpha^3}{3\alpha} + \frac{(1 - \alpha^2)^2}{4} \right), \quad E = \frac{D}{12} (12 - 7\alpha + 5\alpha^3 - 3\alpha^5 + \alpha^7),$$

$$T = \frac{E}{v_c} \approx \frac{D}{12} \left(\frac{1}{v} + \frac{\tau_1}{w} \right) (12 - 7\alpha + 5\alpha^3 - 3\alpha^5 + \alpha^7), \text{ and the expected walking time is } A = \frac{w}{v_w}. \text{ Note that in}$$

the fixed-route system, $\frac{D}{N}$ represents the spacing between adjacent bus routes at the boundary of the region.

3.3.2. Taxi

For comparison, we consider an idealized situation where “chartered” vehicles (e.g., taxi) deliver each passenger directly from its origin to its destination, and we ignore passenger waiting time at the origin. Hence, the user costs only include the travel time from origin to destination. In the $D \times D$ square area, the expected travel distance for each passenger is obviously $E = 2D/3$, and hence the expected travel time per trip is:

$$T = \frac{E}{v} = \frac{2D}{3v}. \quad (14)$$

Here we have assumed that travel speed v equals the vehicle cruising speed (i.e., ignoring the stopping time to pick up the only passenger). The agency costs depend on the total vehicle-distance per hour of operation, Q , and the fleet size, M . We further ignore any vehicle distance

traveled between delivery trips (e.g., the distance traveled to pick up the next passenger)⁵, and hence

$$Q = \lambda D^2 E = \frac{2}{3} \lambda D^3, \text{ and } M = \frac{Q}{v} = \frac{2\lambda D^3}{3v}. \quad (15)$$

Then the objective function for the taxi system is:

$$\text{Min } z = \pi_Q Q + \pi_M M + T. \quad (16)$$

It shall be noted that when $\pi_Q = \frac{\$Q}{\lambda D^2 \mu}$ and $\pi_M = \frac{\$M}{\lambda D^2 \mu}$, the objective function (16)

becomes $\frac{2D}{3v\mu}(\$Qv + \$M + \mu)$, which is independent of the decision variables and demand density.

Also note that the minimum user cost, T , by itself provides an absolute lower bound of the system cost for any transportation mode with speed v .

3.3.3. Discussion

Qualitatively, the proposed flexible transit system could be beneficial because it eliminates the need for passengers to walk to the bus stations. This is desirable because it implies a higher level of service to the passengers. In some circumstances (e.g., transit service during night), the benefit also comes from enhanced passenger safety. In addition, the flexible transit system reduces possible need for infrastructure investment, because there are no specific bus lines or stops. However, these advantages may come at a cost: the total vehicle travel distance (and consequently the passenger riding time) could be higher than that in a fixed-route system, because buses also move laterally in their tubes.

⁵ This assumption and the other simplified assumptions are in favor of the taxi service and hence conservative for our comparison.

As the lateral distance is highly dependent of the number of passengers in the tube, we would expect the flexible-route transit system to be relatively desirable under low demand, while the traditional fixed-route transit system to be desirable under high demand. This intuition is quantitatively verified with the numerical examples in Section 4.

3.4. Numerical analysis

3.4.1. Cost comparison

We compute the optimal system costs of three transit service systems: fixed-route transit, flexible-route transit, and taxi systems, to serve a square area with $D=10$ km and λ varying from 1 to 500 passengers per hour per km^2 . For comparison, we use parameter values that are either realistically assumed or consistent with those in the literature⁶, as shown in Table 3.1.

⁶ Most parameters in Table 1 are taken from Daganzo (2010), except that we use a smaller value for bus deceleration and acceleration time per stop, $\tau_1 = 12$ seconds—per Levinson (1983), $\tau_1 = 11.13$ seconds on average. We also assume that the spacing between adjacent local streets is $s=0.15$ km (www.Wikipedia.com).

Table 3.1 Parameters setting for the numerical examples.

Parameter	s (km)	μ (\$/hr)	τ (sec)	τ_1 (sec)	τ_2 (sec)	v (km/hr)	v_w^7 (km/hr)	δ	$\$Q$ (\$/veh-km)	$\$M$ (\$/veh-hr)
Fixed transit	0.15	20	-	12	1	25	2	0.03	2	40
Flexible transit	0.15	20	13	-	-	25	2	0.03	2	40
Taxi	-	20	-	-	-	25	-	-	2	40

Figure 3.3 plots the optimal cost (per passenger) versus demand density λ for the three systems. The cost of the taxi system is independent of λ . The proposed flexible-route transit system has a lower cost than the fixed-route system when demand is low, while the cost of the fixed-route transit system decreases faster as demand increases. Among the three systems, taxi is desirable under extremely low demand (e.g., $\lambda < 4$ passengers per hour per km^2), while the fixed-route transit system is favorable under high demand. In the middle range when demand is low-to-moderate (e.g., $\lambda = 4 \sim 40$ passengers per hour per km^2), the flexible-route transit system is advantageous. If we realize that the cost functions for the taxi system and for the fixed-route transit system are derived under relatively favorable conditions (e.g., ignoring taxi trips between deliveries and infrastructure investment), then the flexible transit system is probably the best choice for a larger range of demand densities.

⁷ The average walking distance is normally 3 to 5 km/hr but we consider a lower speed to address the discomfort and delay associated with walking.

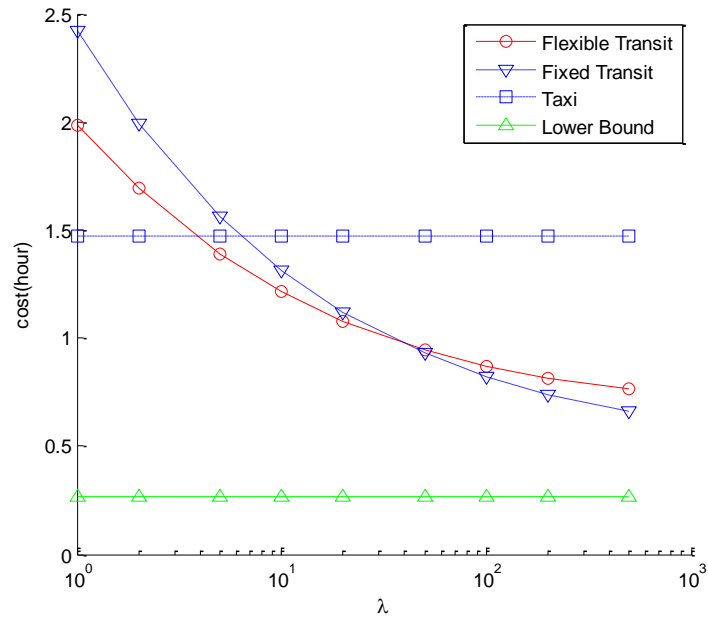


Figure 3.3 System cost versus demand density (log-scale).

Table 3.2 shows the detail of the optimal design for the proposed flexible-route transit system. When travel demand increases, the optimal size of the central square slightly increase, but the optimal number of tubes increases and the optimal headway decreases considerably as expected. The maximum expected bus occupancy increases at a relatively slow rate, and the average bus velocity decreases only moderately. Almost all agency and user cost components (per passenger) decrease monotonically, except that the number of transfers (and hence the associated cost) remains almost the same.

Table 3.2 Network design and cost components for the flexible-route transit system.

λ	α	N	H (hr)	O	v_c	$\$Q$	$\$M$	W	T	$\delta \frac{e_T}{v_w}$	z
1	0.27	3	0.45	25.75	23.45	0.59	0.50	0.47	0.41	0.02	1.99
2	0.31	4	0.35	25.51	23.09	0.48	0.41	0.38	0.41	0.02	1.69
5	0.35	6	0.26	27.89	22.49	0.36	0.32	0.29	0.40	0.02	1.39
10	0.36	8	0.21	34.22	21.95	0.29	0.27	0.24	0.40	0.02	1.22
20	0.35	10	0.15	41.50	21.54	0.25	0.23	0.17	0.40	0.02	1.08
50	0.34	14	0.11	59.53	20.68	0.19	0.19	0.13	0.42	0.02	0.94
100	0.37	19	0.08	64.39	20.18	0.17	0.17	0.09	0.41	0.02	0.87
200	0.36	24	0.06	84.12	19.54	0.15	0.15	0.07	0.43	0.02	0.81
500	0.38	35	0.04	106.40	18.81	0.13	0.13	0.05	0.44	0.02	0.77

For comparison, Table 3.3 shows the optimal design for the fixed-route system. When travel demand increases, the optimal size of the central square and the optimal number of bus tubes both increase. From Tables 3.2-3.3 we see that the flexible transit system seems to require relatively fewer tubes under lower demand and relatively more tubes under high demand. The maximum expected bus occupancy and average bus speed follow similar trends as those of the flexible transit system. All the agency and user cost components (per passenger) decrease with demand except that the in-vehicle travel time increases slightly (due to a lower bus speed v_c and a higher number of stops N).

Table 3.3 Network design and cost components for fixed-route transit.

λ	α	N	H (hr)	O	v_c	$\$Q$	$\$M$	A	W	T	$\delta \frac{e_T}{v_w}$	z
1	0.82	7	0.65	4.54	23.57	0.38	0.32	0.37	0.69	0.29	0.02	2.42
2	0.83	9	0.52	5.73	23.23	0.30	0.25	0.29	0.55	0.29	0.02	1.99
5	0.85	11	0.38	7.62	22.72	0.22	0.19	0.22	0.40	0.30	0.02	1.56
10	0.86	14	0.30	9.67	22.27	0.17	0.15	0.18	0.31	0.30	0.02	1.31
20	0.88	17	0.24	12.01	21.74	0.13	0.12	0.15	0.25	0.31	0.02	1.12
50	0.90	21	0.17	15.99	21.03	0.09	0.09	0.12	0.17	0.32	0.02	0.93
100	0.92	24	0.13	19.38	20.45	0.07	0.07	0.10	0.13	0.33	0.02	0.82
200	0.93	29	0.10	24.53	19.79	0.06	0.06	0.09	0.10	0.34	0.02	0.74
500	0.96	33	0.07	32.05	19.01	0.04	0.04	0.08	0.07	0.35	0.02	0.66

As discussed in Section 3.3, for all λ , the agency costs in the flexible transit system (Q and M) are larger than their counterparts in the fixed transit system, and each passenger spends more time (T) in the vehicle, because the buses in the flexible system travel extra distances to pick up and drop off passengers. In the flexible transit the ratio of longitudinal over lateral travel over distance varies approximately between 0.54 and 1.67. The advantage of the flexible system is that the passenger walking time (A) is completely eliminated, while the waiting time (W) is also reduced.

For the taxi system, we can easily compute from (16)–(18) that the optimal system cost components are $Q=0.67$, $M=0.53$, $T=0.27$, and hence the system cost per passenger is $z=1.47$. The agency costs in the flexible transit system (Q and M) are much smaller than that of the taxi system (even under our favorable assumptions), while the taxi system incurs the smallest in-vehicle travel time.

3.4.2. Sensitivity analysis

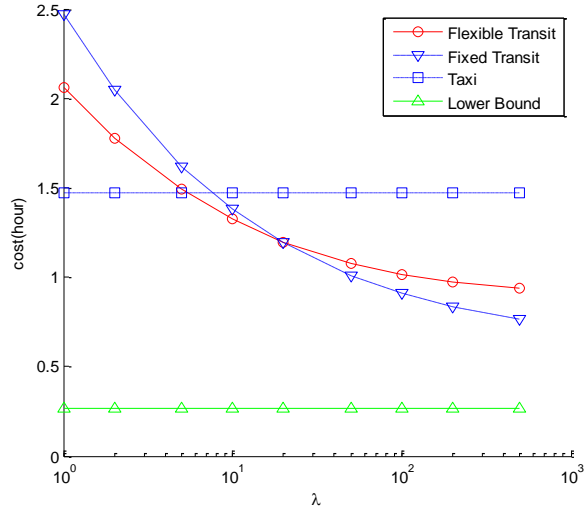
In Section 4.1, we assumed that $\tau_1 = 12$ seconds (Levinson, 1983) for both fixed and flexible transit systems. In Daganzo (2010) a more conservative value of $\tau_1 = 30$ seconds is assumed. Intuitively, longer bus delay per stop will have a direct impact on the total number of buses in operation and the expected travel time per passenger. Such impact will be especially significant for the flexible transit system, because it makes a stop for each passenger visit. To this end, we conduct a sensitivity analysis to see how the three systems perform under $\tau_1 = 30$ seconds, and the results are shown in Figure 3.4(a). We can see that all curves follow the same trend as those in Figure 3.3, but the optimal costs for the flexible- and the fixed-route transit systems have increased (more so for the flexible system). Although the range of suitable demand for the flexible system has shrunk, we nevertheless notice that the flexible transit system remains the best choice when $\lambda = 5 \sim 20$ passengers per hour per km^2 .

Under certain circumstances (e.g., adverse weather, night time), passenger walking is associated with significant discomfort or passenger safety becomes a major concern. This situation can be addressed by a dramatic reduction in walking speed v_w . We consider the extreme case when v_w reduces to 0.1 km/hour, and the cost curves are shown in Figure 3.4(b). The total cost for the fixed transit system increases dramatically, mainly due to the increase in walking time (A). The flexible transit system, in contrast, only bears a slight increase in the total cost. The cost for the taxi system remains unchanged. In this case, the flexible transit system becomes favorable for a much larger range of customer demand densities.

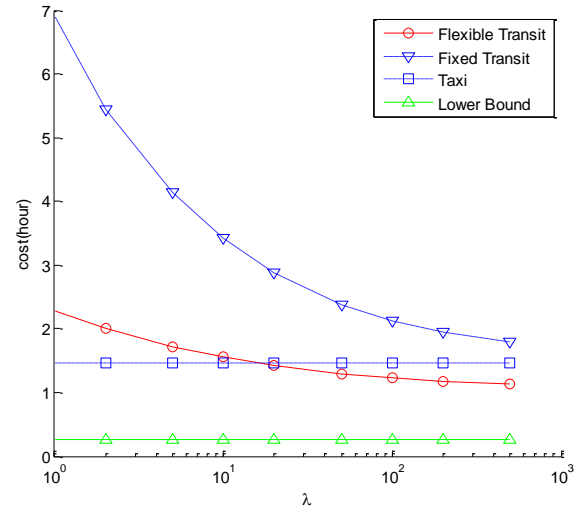
Figure 3.4(c) shows the system cost curves under a higher walking speed, 3.0 km/hour. Even with this walking speed the proposed system shows a lower cost for some demand densities. We would like to highlight that the flexible transit system is particularly targeting

situations in which passenger walking is inconvenient (e.g. severe weather condition, night time, or unsafe area). Also note that the actual taxi cost probably much higher than the conservative one shown in this figure; the actual demand range for the flexible system to be optimal is probably much larger.

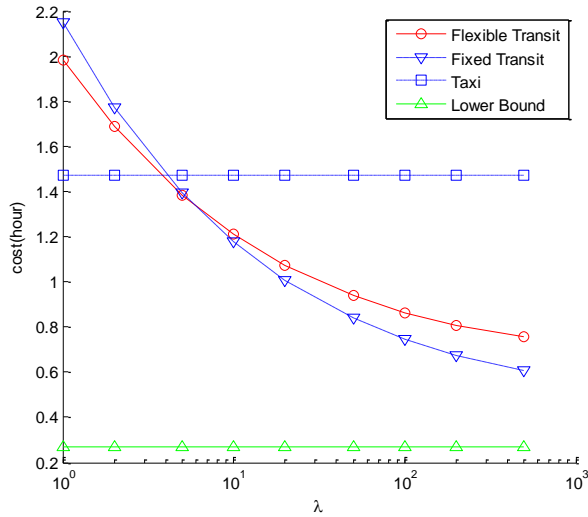
In the base case the expected waiting time and in-vehicle travel time are assumed to be the same to the passengers. In reality, waiting time is sometimes considered to be less convenient. To study this effect, we consider a case where one unit of waiting time is 1.8 times as costly as in-vehicle travel time. The resulting cost curves are plotted in Figure 3.4(d). Since the relative significance of waiting time in the fixed and flexible route systems are approximately the same, the relative comparison across different systems is similar to that of the base case.



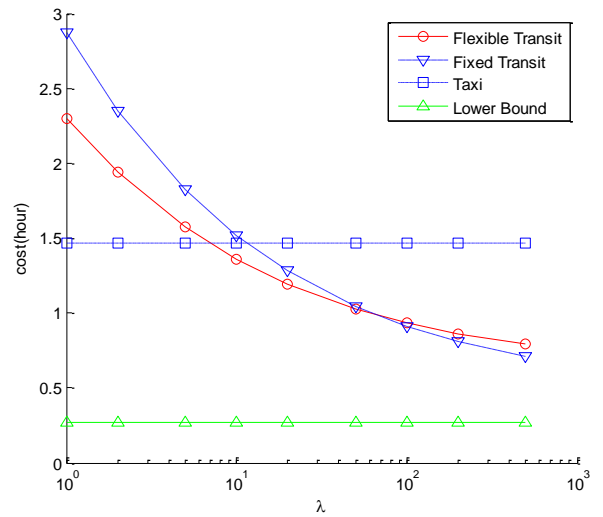
(a) $\tau = 30$ seconds



(b) $v_w = 0.1$ km/hr



(c) $v_w = 3.0$ km/hr



(d) (waiting time) = 1.8 (in-vehicle time)

Figure 3.4 System cost versus demand density: (a) long bus dwell time; (b) inconvenient walking; (c) higher walking speed; (d) higher weight of waiting time.

3.5. Conclusion

This chapter proposed a structured flexible-route transit system in which buses pick up or drop off passengers within their predetermined service areas (i.e., bus tubes) with no specific routes. Collectively, the bus tubes form a “grand” structure, which includes a grid tube network that provides double coverage to passengers in the central part of the city, and a hub-and-spoke tube network that provides a single coverage in the peripheral part. We analyzed all relevant agency and user costs associated with the system design, and optimized the system by determining the layout of the grand network, the bus tube size, and the bus headway.

The proposed transit system is shown to have the potential to provide a higher level of service to the passengers, mainly by eliminating walking time to and from bus stations. This is particularly desirable under situations where concerns over pedestrian safety are high (e.g., at night or under adverse weather). Quantitative analysis of the proposed system shows that under low-to-moderate passenger demand the system incurs lower cost than other conventional counterparts such as the fixed-route transit system and the chartered taxi system. In practice, the proposed flexible transit system is also easy to implement. The grand structure can be used to guide the tube layout while details of the local streets and neighborhoods are considered. Existing spatial partitioning models (e.g. the disk models in Ouyang and Daganzo, 2006; Ouyang, 2007) can be potentially adapted to help align bus tubes and adjust the size.

Future research could be conducted to explore other network structures (e.g., a ring-radial network) or any hybrid combination of them (e.g. multi-hub-and-spoke). Other shapes of the service region (e.g., rectangle, circle) or heterogeneous demand distributions (e.g., monocentric or multicentric) could be interesting to analyze, although these considerations may add extra complexity to the model.

Chapter 4

Continuum Approximation Approach to Bus Network Design under Spatially Heterogeneous Demand⁸

4.1. Introduction

When designing a public transit system, one would ideally lay-out routes and establish service frequencies in ways that accommodate travel demand at minimum generalized cost to the system's users and to its operating agency. Most efforts in this realm have relied upon mathematical programming models and numerical solution methods (e.g. Kuah and Perl, 1989; Uchimura, et al, 2002; Lee and Vuchic, 2005; Fan and Machemeh, 2006). These computational burdens often limit the alternatives that are considered in the design process; i.e., the process may consist of only a narrow range of numerical experiments. The resulting insights on how to best design and manage a transit system may equally be narrow.

In efforts to obtain transit designs that are more general and insightful in nature, models have been developed that require fewer and less detailed inputs. Simplicity in this regard is achieved by expressing the inputs and outputs as continuous quantities; e.g. transit routes and the stops along those routes are specified in terms of the physical spacings that separate them, and demand

⁸ This chapter has been adopted from “Ouyang, Y., Nourbakhsh, S.M. and Cassidy, M. (2014) “Continuum approximation approach to transit network design under heterogeneous demand.” *Transportation Research Part B*, 68: 333-344.”.

for transit travel is expressed as a density, rather than in detailed O-D tables (e.g. Newell, 1973; Daganzo, 1992). In this way, the network design problem for an entire city often conveniently reduces to one that involves only a few key decision variables, such as route and stop spacing and vehicle headway. Solutions to these simpler models can often be obtained analytically, and the resulting insights can be ideal for high-level decision-making. High-level decision-making refers to the strategic and tactical decisions that do not change in the short term (e.g. route and stop spacing and headway).

The simplicity afforded by the above models comes with downsides as well. Of particular concern, these models yield closed-form solutions by treating travel demand as homogeneous over both time and space. This is rarely the case in real settings, however. Fortunately, temporal variations in demand often span periods that are much longer than a vehicle's headway. Thus, temporal demand variations can often be accommodated relatively easily by operating fleets in flexible manners; e.g. by dispatching buses at frequencies that vary with time of day (Daganzo, 2010b).

Spatial variations in demand can be more problematic. Variations of this kind often dictate that transit networks display heterogeneous layouts in which some neighborhoods have denser routes and stops than do others. Though Daganzo (2010) offers qualitative design principles for heterogeneous spatial demand, a detailed methodological framework for these cases has not previously been proposed.

The erstwhile absence of a suitable methodology is not surprising. After all, it can be challenging to formulate a design framework that relies on only a few (neighborhood-level) design variables, and that can still produce suitable heterogeneous network configurations. The

challenge arises because: route and station spacings in a neighborhood depend not only on the local demand generated by that neighborhood, but also on the numbers of en-route passengers that travel through the neighborhood from elsewhere. Moreover, the share of a system's total travel demand that is garnered by a neighborhood route depends in turn upon the route and stop spacings in that neighborhood and in others. And where network layout varies over space, transitions between distinct layouts in adjacent neighborhoods will be needed to ensure that all passengers enjoy connectivity between their origins and their destinations. A transition may require that passengers transfer between vehicles that serve distinct routes.

The above challenges are addressed in the present chapter. We offer a continuous approximation (CA) framework for bus-system design when travel demand varies smoothly over space.⁹ The methodology produces a hybrid layout of routes, but one that is distinct from that of Daganzo (2010a). Ours features a grid of both, “main” bus routes that are placed at relatively large spacings in those neighborhoods where demand is low, and one or more grids of closely-spaced “local” routes that cover neighborhoods of high demand. An illustration is given in Fig 4.1a. In general we can have more than one “local” routes (Fig 4.1b).

⁹ We make this assumption mainly because (i): the error due to such location approximation might be comparable to the data “aggregation” process that most discrete model requires and (ii): a detailed investigation of the methodology shows that the CA cost formulas are largely linear in the demand density, while the CA method essentially is taking local order-zero approximations. Hence, CA method may not be too bad.

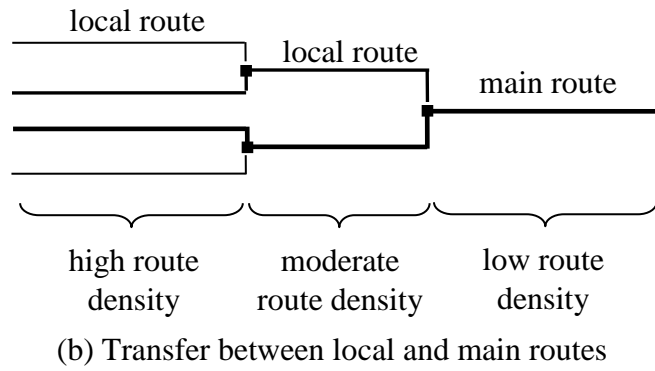
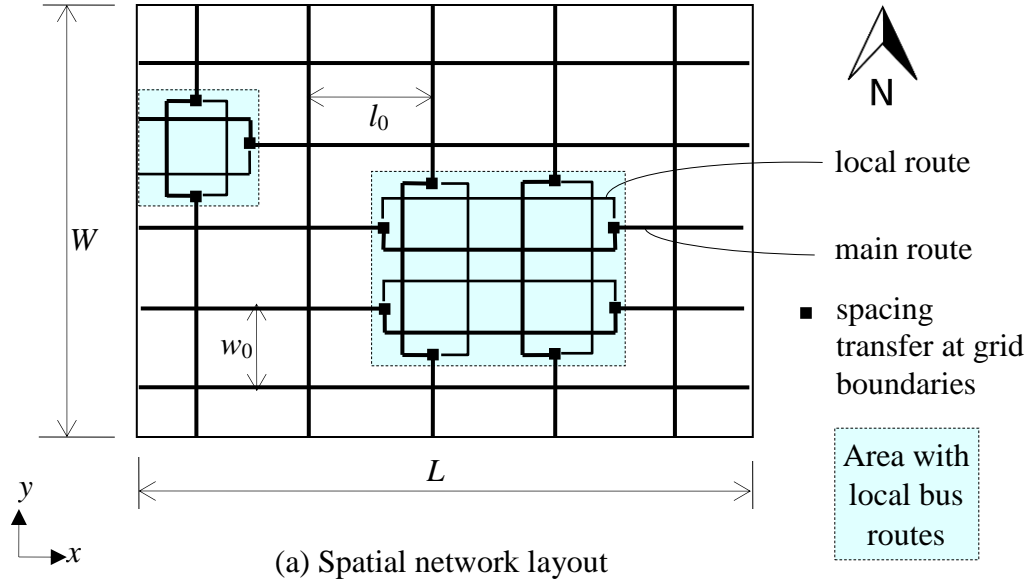


Figure 4.1 General scheme of a heterogeneous transit network.

Main and local buses travel back and forth along the lengths of their assigned routes. Main routes span the entire city; i.e., each has a length of either L or W , depending upon its orientation, as shown in Fig 1a. At the boundaries between distinct grids, routes either bifurcate, or two routes converge to one, depending upon the travel direction. Examples of this geometry are illustrated in Fig 1b. To clarify ideas, consider a main route serving travel in the westbound

(leftward) direction, and note from Fig 1b how the main route bifurcates at each grid's boundary. The main route continues along a westerly path while a new, local route emerges at each bifurcation point and thereafter runs parallel to the main route. (The reader can also use Fig 1b to envision how, in the eastbound travel direction, two routes converge at each grid's boundary.) A bifurcation (or convergence) point marks an endpoint for a local route, and passengers can transfer between two routes at these points.

To achieve the geometries of Fig 1, we require that the spacings between parallel routes conform to a power-of-two scale (Roundy, 1985). This scale is known from the inventory control literature to help synchronize production schedules at very small loss of optimality. We borrow this concept to design spatially heterogeneous bus networks that provide seamless alignment of bus routes in spite of varying spacings.

The remainder of this chapter is organized as follows. The modeling framework and iterative solution algorithm are described in the following section. The numerical experiments are presented in section 4.3. Implications and future research directions are discussed in section 4.4.

4.2. Methodology

Consider a rectangular city of length L in the E-W travel directions (along the x axis), and width W in N-S directions (y axis); see again Fig 1a. The demand for bus travel from a city's neighborhood that encompasses coordinates (x_1, y_1) to another neighborhood encompassing (x_2, y_2) has a density $\delta(x_1, y_1, x_2, y_2)$ that has units of passengers per time per square area and that varies smoothly over all its arguments.

Assume that the city has a very dense grid of E-W and N-S streets, and that any of these can serve as bus routes if needed. Recall that main routes begin and end on the city's boundaries, while local routes can start and end anywhere within the city. Bus stops are placed at the intersections of the routes, such that the stop spacing along an E-W route is the route spacing for the N-S routes that cross it, and vice-versa. A route's service (or catchment) area is measured by its spacing with the parallel adjacent routes. These spacings are assumed to be small relative to the city's dimensions W and L , such that many buses simultaneously travel back and forth along the city's collection of routes.

Buses are assumed to have the following: a common cruising speed, v , a common time spent dwelling at a stop to load and unload passengers, τ , and sufficient passenger-carrying capacities to accommodate their demand. We will further assume that buses: stop to serve passengers at every bus stop along their routes; and operate at a headway, H , that is common across all routes. This simplified assumption raises two concerns that are related to fleet/crew management: (i) how to stabilize bus arrival fluctuations so as to allow synchronization across the routes; (ii) how to reconcile different round-trip cycle time across routes (e.g., by sharing buses around multiple local routes). However, this latter assumption regarding H opens the door to a simple way of modeling passenger route-choice behavior, as described next.

4.2.1. Assumed Passenger Behavior

Similar to Daganzo (2010a), we assume that passengers (i) walk at speed v_w to and from the stops closest to their origins and destinations; (ii) travel onboard between these stops as directly as, and with the fewest number of transfers, possible; and (iii) randomly choose their trip start

times and whenever there is a tie among multiple alternatives, randomly choose their bus routes as well. Assumption (iii) is reasonable because either choice of initial travel direction will yield the same trip experience given the common H , and passengers are likely to break ties arbitrarily.

The number of transfers required of a passenger will, of course, depend upon her origin and destination. We will assume that for each trip, one transfer will occur at the intersection of two perpendicular bus routes so that the passenger can change travel direction. We will refer to this as a “directional transfer,” and our assumption that one transfer of this kind occurs per trip is conservative because it ignores the small possibility that a trip’s origin and destination will sometimes lie along or near a single route. Some trips may entail one or more additional transfers that occur at the peripheral bounds of distinct grids (refer again to Fig 1b) and which we refer to as “spacing transfers.” As an illustration of a worst-case scenario for a system similar to Fig 1a, consider a passenger who: starts her trip within a local grid; performs a spacing transfer to a main route; and after performing a directional transfer, ultimately reaches her destination via yet another spacing transfer to a final route that lies within a second local grid.

4.2.2. Power-of-Two Scheme

The design problem consists of jointly determining the grid layouts (i.e., route and stop spacings in all neighborhoods) and the H that minimize the generalized cost to the bus agency and the users. Route and stop spacings would ideally vary gradually over space. However, our decision to enforce the power-of-two concept (Roundy, 1985) means that, at the cost of small loss in optimality, the spacings between a neighborhood’s N-S routes, $l(x,y)$, and between its E-W routes, $w(x,y)$, occur on a discrete exponential scale; i.e.,

$$l(x, y) = l_0 / 2^{k(x, y)}, w(x, y) = w_0 / 2^{k(x, y)},$$

where l_0 and w_0 are the base spacings between N-S and E-W routes, respectively, and $k(x, y) \in \{0, 1, 2, \dots\}$. Thus, the network layout decisions regarding $\{l(x, y)$ and $w(x, y)\}$ for all (x, y) can be equivalently determined from l_0 , w_0 and $\{k(x, y)\}$ for all (x, y) . Since we assume that all system parameters vary slowly over space, we further assume that every change in route spacing involves the merging (or bifurcation) of two routes and no more; i.e., the value of $k(x, y)$ does not have sharp jumps with respect to either x or y .

4.2.3. Continuum Approximation

We compute the expected rate per area per time that passengers start and end their trips at a stop located at (x, y) , $D_{\text{start}}(x, y)$ and $D_{\text{end}}(x, y)$, from the slow-varying demand functions between the stop at (x, y) and all others in the city at (\bar{x}, \bar{y}) ; i.e.,

$$D_{\text{start}}(x, y) = \int_{\bar{y}=0}^W \int_{\bar{x}=0}^L \delta(x, y, \bar{x}, \bar{y}) d\bar{x} d\bar{y}, \quad D_{\text{end}}(x, y) = \int_{\bar{y}=0}^W \int_{\bar{x}=0}^L \delta(\bar{x}, \bar{y}, x, y) d\bar{x} d\bar{y}. \quad (1)$$

To determine the approximate rates per unit area per time that passengers perform directional transfers at (x, y) , $D_{\text{d_transf}}(x, y)$, one may first visualize: all trips with origins that lie within the elementally thin horizontal swath in Fig 2 and that have destinations within the vertical swath;

and all analogous trips with origins in the vertical swath and destinations in the horizontal one.

Consideration shows that:

$$D_{\text{d_transf}}(x, y) = \frac{1}{2} \int_{\bar{y}=0}^W \int_{\bar{x}=0}^L [\delta(\bar{x}, y, x, \bar{y}) + \delta(x, \bar{y}, \bar{x}, y)] d\bar{x} d\bar{y}, \quad (2)$$

where the coefficient $\frac{1}{2}$ reflects the passengers' arbitrary choice of initial travel direction, as per assumption (iii) in Section 2.1. Note that the integrands of (2) pertain to the directional transfers represented by the arrows in Fig 4.2. Given the route spacings $l(x, y)$ and $w(x, y)$, the rates that passengers start trips, end trips and perform directional transfers at the stop per unit time are approximately $l(x, y)w(x, y)D_{\text{start}}(x, y)$, $l(x, y)w(x, y)D_{\text{end}}(x, y)$, and $l(x, y)w(x, y)D_{\text{d_transf}}(x, y)$, respectively.

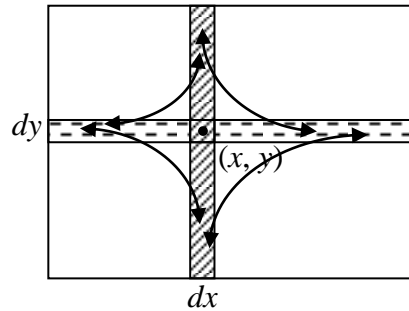


Figure 4.2 Directional transfer near point (x, y) .

From assumptions (i) – (iii), we can compute the expected fluxes (per time-distance) of onboard passengers passing (x, y) in all directions. We denote these as $F_w(x, y)$, $F_E(x, y)$, $F_S(x, y)$ and $F_N(x, y)$ for the westbound, eastbound, southbound and northbound

directions, respectively. To visualize how these computations are performed, consider first the eastbound rates per area, $F_E(x, y)$. Consideration shows that:

$$F_E(x, y) = f_E^1(x, y) + f_E^2(x, y), \quad (3a)$$

where:

$$f_E^1(x, y) = \frac{1}{2} \int_{x_1=0}^x \left[\int_{x_2=x}^L \int_{y_2=0}^W \delta(x_1, y, x_2, y_2) dy_2 dx_2 \right] dx_1, \quad f_E^2(x, y) = \frac{1}{2} \int_{x_2=x}^L \left[\int_{x_1=0}^x \int_{y_1=0}^W \delta(x_1, y_1, x_2, y) dy_1 dx_1 \right] dx_2. \quad (3b)$$

The coefficients of $\frac{1}{2}$ again reflect the passengers' arbitrary choice of initial travel direction; the first term, $f_E^1(x, y)$, pertains to trips that originate within the elementally thin horizontal slice in Fig 3a that are destined for the wider, shaded rectangular region; and the second term, $f_E^2(x, y)$, pertains to the analogous O-D patterns of Fig 3b.

The westbound flux, $F_W(x, y)$, can be obtained by swapping the $\delta(x_1, y, x_2, y_2)$ and $\delta(x_1, y_1, x_2, y)$ in (3a)-(3b) with $\delta(x_2, y_2, x_1, y)$ and $\delta(x_2, y, x_1, y_1)$, respectively. Thanks to symmetry, flux $F_S(x, y)$ is obtained by swapping all the x and y symbols, and swapping all the L and W in the formula for $F_W(x, y)$. Flux $F_N(x, y)$ is obtained by similarly swapping symbols in (4.3a)-(4.3b). Moreover, by knowing the maximum flux we can obtain the bus capacity.

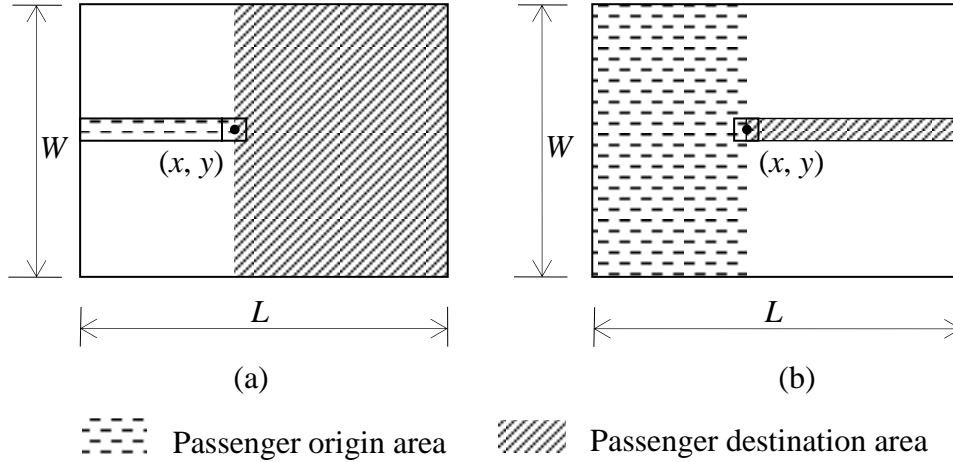


Figure 4.3 Directional flow of onboard passengers $F_E(x, y)$ near location (x, y) .

Finally, we define a set of functions for potential spacing transfers near location (x, y) . We use the word “potential” because a portion of the passengers onboard a bus will make a spacing transfer only if there exists a bifurcation (or convergence) point. Two possibilities arise: some passengers will transfer out of a local line while still near their origins, before making a directional transfer; and some will transfer onto a local line when finally near their destinations, after having performed a directional transfer. For example, consider the case in which a route convergence occurs in the eastbound direction at (x, y) . Approximately half of the on-board passengers traveling along the eastbound local route will perform a spacing transfer at (x, y) .¹⁰ The passengers who transfer will be those whose origins and destinations are like those illustrated in Fig 3a. Hence, the transfer number is approximately $\frac{1}{2} f_E^1(x, y)$. Those traveling in the westbound direction will face a route bifurcation, and about half of those on-board

¹⁰ The assumption here is that the onboard passengers have randomly boarded the two parallel bus routes independent of their origins and destinations. This estimate is conservative because it ignores the small probabilities that a passenger who starts her trip on a main route may complete her trip without performing a spacing transfer even if this passenger passes some dense network areas.

passengers will perform a spacing transfer to move onto the local route. The origin-destination pattern of each of those transferring passengers is like that in Fig 3b, except that the origin/destination areas are swapped. The number of these transfers is approximately $\frac{1}{2} f_W^2(x, y)$.

If route convergence occurs instead in another direction, be it westbound, northbound or southbound, the total number of spacing transfers can be obtained by exploiting the symmetry in the problem to obtain a formulation similar to (3b). Ideally, the existence of a convergence (or a bifurcation) would be indicated by a discrete change in the value of $k(x, y)$ along the x and y directions. This discrete change is generally difficult to compute without looking at the overall shape of the $k(x, y)$ function. To simplify things, we temporarily treat $k(x, y)$ as a continuous function, and prorate the changes by its “local” partial derivatives. We then use the following formula to approximate the rate of spacing transfers per unit area for all four travel directions:

$$D_{s_transf}(x, y) \approx \frac{1}{2} \left(f_E^2(x, y) + f_W^1(x, y) \right) \left[\frac{\partial}{\partial x} k(x, y) \right]^- + \frac{1}{2} \left(f_E^1(x, y) + f_W^2(x, y) \right) \left[\frac{\partial}{\partial x} k(x, y) \right]^+ \\ + \frac{1}{2} \left(f_N^2(x, y) + f_S^1(x, y) \right) \left[\frac{\partial}{\partial y} k(x, y) \right]^- + \frac{1}{2} \left(f_N^1(x, y) + f_S^2(x, y) \right) \left[\frac{\partial}{\partial y} k(x, y) \right]^+ \quad (4)$$

where $[\cdot]^+ = \max\{0, \cdot\}$ and $[\cdot]^- = -\min\{0, \cdot\}$. This approximation is conservative, since it captures all local variations of a continuous function $k(x, y)$, including those that do not actually contribute to a discrete change in $k(x, y)$ value (after rounding).

4.2.4 Agency Costs

We are now in a position to estimate costs. Consider three components of agency cost that are due to: buses stopping to load and unload passengers, denoted G ; and the total vehicular distance

traveled and vehicular time expended, denoted V and M , respectively. These costs in the neighborhood of (x, y) can be expressed per unit area per time as follows:

$$G(x, y) = 4c_s l^{-1}(x, y)w^{-1}(x, y)H^{-1} = c_s 4^{k(x, y)+1} l_0^{-1} w_0^{-1} H^{-1}, \quad (5)$$

$$\begin{aligned} V(x, y) &= 2c_v H^{-1} \left[l^{-1}(x, y) + w^{-1}(x, y) + \frac{w_0^{-1}}{2} \left\{ \left[\frac{\partial}{\partial x} k(x, y) \right]^- + \left[\frac{\partial}{\partial x} k(x, y) \right]^+ \right\} + \frac{l_0^{-1}}{2} \left\{ \left[\frac{\partial}{\partial y} k(x, y) \right]^- + \left[\frac{\partial}{\partial y} k(x, y) \right]^+ \right\} \right] \\ &= c_v 2^{k(x, y)+1} H^{-1} \left(l_0^{-1} + w_0^{-1} + \frac{w_0^{-1}}{2} \left\{ \left[\frac{\partial}{\partial x} k(x, y) \right]^- + \left[\frac{\partial}{\partial x} k(x, y) \right]^+ \right\} + \frac{l_0^{-1}}{2} \left\{ \left[\frac{\partial}{\partial y} k(x, y) \right]^- + \left[\frac{\partial}{\partial y} k(x, y) \right]^+ \right\} \right) \end{aligned} \quad (6)$$

,

$$M(x, y) \approx \frac{c_M}{c_v} V(x, y) \left[v^{-1} + \frac{2\tau}{l(x, y) + w(x, y)} \right], \quad (7)$$

where the coefficients c_s , c_v and c_M are the unit operation costs per bus stop, per bus mile and per bus hour, respectively. Equation (5) underscores that the spatial density of bus stops in the neighborhood of (x, y) is $l^{-1}(x, y)w^{-1}(x, y)$, and that buses traveling in each of the four directions will stop at (x, y) once within a single headway, H . Equation (6) underscores that in each H , the total distance traveled by the four directional buses per unit area is $2[l^{-1}(x, y) + w^{-1}(x, y)]$. Finally,

(7) incorporates the locally-approximated commercial speed of buses, $v \left[1 + \frac{2\tau v}{l(x, y) + w(x, y)} \right]^{-1}$,

which factors-in the time spent at the bus stop.

4.2.5 User Costs

We consider four components of user cost that are due to: the time that a user spends walking from an origin to the first bus stop, and from the final bus stop to a destination, A ; the out-of-vehicle waiting times at stops, including those at transfers, W ; the in-vehicle travel times, $IVTT$, and the extra penalties associated with transfers, Q . Equations (8)-(11) give these user costs per unit area per unit time in the neighborhood of (x, y) .

$$A(x, y) = \frac{1}{4v_w} [l(x, y) + w(x, y)] [D_{\text{start}}(x, y) + D_{\text{end}}(x, y)], \quad (8)$$

$$W(x, y) \approx \frac{H}{2} [D_{\text{start}}(x, y) + D_{\text{d_transf}}(x, y) + D_{\text{s_transf}}(x, y)], \quad (9)$$

$$IVTT(x, y) = \frac{1}{v} (F_W(x, y) + F_E(x, y) + F_S(x, y) + F_N(x, y)) + \tau \left(\frac{F_E(x, y) + F_W(x, y)}{w(x, y)} + \frac{F_N(x, y) + F_S(x, y)}{l(x, y)} \right), \quad (10)$$

$$Q(x, y) = \theta [D_{\text{d_transf}}(x, y) + D_{\text{s_transf}}(x, y)]. \quad (11)$$

where θ is the extra penalty (in the unit of time) associated with the onerousness of each transfer in addition to the time spent waiting at a transfer stop. Equation (9) assumes that bus headways are maintained without variation and are not synchronized across intersecting routes, such that the expected passenger out-of-vehicle wait time at all stops is $H/2$, as in Daganzo (2010a), Nourbakhsh and Ouyang (2012), and Ouyang et al. (2014). The first term in (10) pertains to the times that onboard passengers in all travel directions collectively spend while the buses each move a unit distance at the cruise speed, v . The second term in (10) accounts for the expected

times that these passengers collectively spend dwelling at stop (x, y) while their buses load and unload other passengers.

4.2.6. Network Design

An average value of time, μ , is used to convert the agency's monetary cost to an equivalent user cost measured in units of time. The design problem below minimizes the expected generalized cost to the agency and to the users per unit time by determining optimal values of the decision variables and functions: H and $l(x,y)$, $w(x,y)$ for every neighborhood (x, y) ; or equivalently, H , l_0 , w_0 and $k(x, y)$ for all (x, y) .

$$\text{Min } \int_{y=0}^W \int_{x=0}^L Z(x, y) dx dy \quad (12a)$$

$$\text{s.t. } l(x, y) = l_0 / 2^{k(x,y)}, \text{ for all } (x, y) \quad (12b)$$

$$w(x, y) = w_0 / 2^{k(x,y)}, \text{ for all } (x, y) \quad (12c)$$

$$k(x, y) \in \{0, 1, 2, \dots\}, \text{ for all } (x, y) \quad (12d)$$

$$H \geq 0. \quad (12e)$$

where $Z(x, y) = \frac{1}{\mu} [G(x, y) + V(x, y) + M(x, y)] + [A(x, y) + W(x, y) + IVTT(x, y) + Q(x, y)]$ is the local generalized cost per area per time.

For any given l_0 , w_0 and H , the $Z(x, y)$ can generally be optimized as a function of $k(x,y)$ in each neighborhood of (x,y) . Again, we first treat $k(x,y)$ as a continuous function. Close

examination of the cost components of $Z(x,y)$ shows that this function can be written in the following form:

$$Z(x, y) = \Phi(x, y) \cdot 4^{k(x,y)} + \Gamma(x, y) \cdot 2^{k(x,y)} + \Pi(x, y) \cdot 2^{-k(x,y)} + \Psi(x, y) + \left[\frac{H}{2} + \theta \right] D_{s_transf}(x, y), \quad (13)$$

where the coefficients for each (x, y) are

$$\Phi(x, y) = \frac{4}{\mu} (c_s + \tau c_M) H^{-1} l_0^{-1} w_0^{-1},$$

$$\Gamma(x, y) = \frac{2}{\mu} (c_V + \frac{c_M}{v}) H^{-1} (l_0^{-1} + w_0^{-1}) + \tau \left(\frac{F_E(x, y) + F_W(x, y)}{w_0} + \frac{F_N(x, y) + F_S(x, y)}{l_0} \right),$$

$$\Pi(x, y) = \frac{1}{4v_w} (l_0 + w_0) [D_{start}(x, y) + D_{end}(x, y)],$$

$$\Psi(x, y) = \frac{H}{2} D_{start}(x, y) + \left(\frac{H}{2} + \theta \right) D_{d_transf}(x, y) + \frac{1}{v} (F_W(x, y) + F_E(x, y) + F_S(x, y) + F_N(x, y)).$$

One can readily verify that, given H , l_0 and w_0 , the first four terms of (13) constitute a convex function of $2^{k(x,y)}$ and hence have a unique optimizer. Given the monotonicity of $2^{k(x,y)}$, the optimal solution $k(x,y)$ is also unique. The last term of (13) is a function of the local gradients of $k(x,y)$ but not the value of $k(x,y)$ itself, as can be verified by referring back to (4). We can therefore obtain a lower bound of (13) by finding $k^*(x, y)$ that optimizes only the first four terms. An upper bound can also be obtained by computing the partial derivatives of $\{k^*(x, y), \nabla(x, y)\}$ and plugging them into all five terms of (13). The integration of this upper bound across the entire city gives a feasible solution to (12), which is also near-optimum if spacing transfers contribute to only a small part of the generalized cost. Finally, we can round $\{k^*(x, y), \nabla(x, y)\}$ to the nearest integer to obtain the final design.

Now we are ready to incorporate the above solution approach into an algorithm that can be used to obtain a city-wide design. We describe the proposed algorithm below in pseudo-code form.

Step 0: Initialize the decision variables: H , l_0 and w_0 , and divide the city into many small cells. Each cell is centered on an (x, y) and the cells collectively cover the entire city.

Step 1: For each cell, compute the average passenger demand (potential) functions

$$D_{\text{start}}(x, y), D_{\text{end}}(x, y), D_{\text{d_transf}}(x, y), D_{\text{s_transf}}(x, y), \quad \text{and} \quad \text{the} \quad \text{onboard} \quad \text{fluxes},$$

$$F_W(x, y), F_E(x, y), F_S(x, y) \text{ and } F_N(x, y).$$

Step 2: Again for each cell, solve the local optimization problem; i.e., determine an integer variable $k(x, y)$ that minimizes the first four terms of $Z(x, y)$ in (13). Calculate the total cost by summing the $Z(x, y)$ value across all cells.

Step 3: Repeat Step 2 to obtain a new total cost after perturbing the triplet $\langle H, l_0, w_0 \rangle$ to $\langle H + \Delta_H, l_0 + \Delta_l, w_0 + \Delta_w \rangle$ for some small $\Delta_H, \Delta_l, \Delta_w$, and numerically compute the gradients with respect to $\langle H, l_0, w_0 \rangle$. If the gradient is within a specified tolerance, go to Step 4; otherwise, apply standard line search methods such as Newton-Raphson to find a step size that updates $\langle H, l_0, w_0 \rangle$, and go to Step 1.

Step 4: Implement the network design based on the best solution. Evaluate the actual agency and user costs based on the actual network design and assumed user behavior described in Section 2.1.

4.3. Numerical results

We next evaluate our iterative model by applying it to a square city with travel demands that are invariant to time, but that vary over space. Four distinct spatial demand distributions are explored: (I) a mono-centric city, where both the origins and the destinations of transit trips are clustered in the city center; (II) a twin city, where the O-D demand is clustered much as before, except that now the clusters occur in two adjacent regions near the city center; (III) an asymmetric mono-centric city, where demand is clustered near a city boundary; and (IV) a commuter city, where a cluster of trip origins and a separate cluster of destinations occur in two distinct centers. For all four cases, the demand densities follow the general function below:

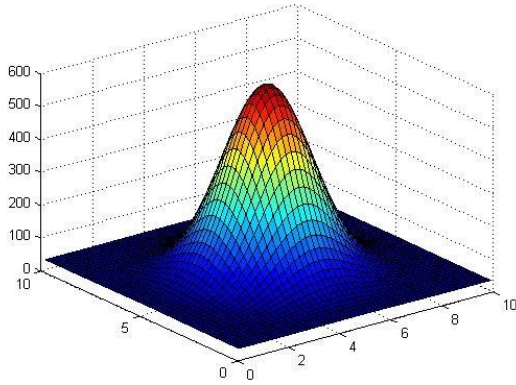
$$\delta(x_1, y_1, x_2, y_2) = \prod_{i=1}^2 \left(a_1 + a_2 \sum_{j=1}^2 \exp \left[-(a_{3j}x_i - b_{ij})^2 - (a_{4j}y_i - \bar{b}_{ij})^2 \right] \right), \quad (14)$$

with parameters that are specified in Table 4.1 for each case. These tabulated values were set so that the city's total rate of transit trip-making is the same (10,000/hr) in each of the four cases.

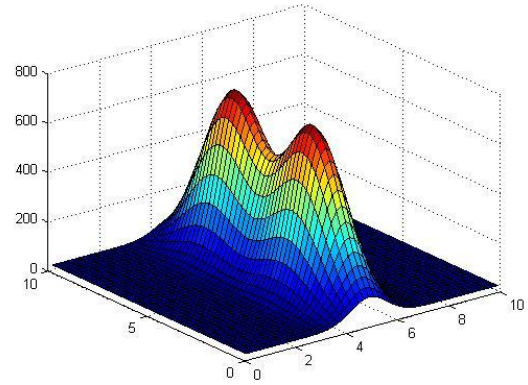
Table 4.1 Demand distribution parameters in cases (I)-(IV).

Case	Parameters													
	a_1	a_2	a_{31}	a_{32}	a_{41}	a_{42}	b_{11}	b_{12}	b_{21}	b_{22}	\bar{b}_{11}	\bar{b}_{12}	\bar{b}_{21}	\bar{b}_{22}
(I): Mono-centric city	0.0016	0.065	0.5	0	0.5	0	2.5	0	2.5	0	2.5	0	2.5	0
(II): Twin city	0.0030	0.12	1	1	0.5	0.5	1.5	2.5	2.5	3.5	1.5	3.5	2.5	2.5
(III): Asymmetrically sprawled city	0.0021	0.081	0.5	0	0.5	0	1	0	1	0	1	0	1	0
(IV): Commuter city	0.00044	0.070	0.5	0	0.5	0	1	0	4	0	4	0	1	0

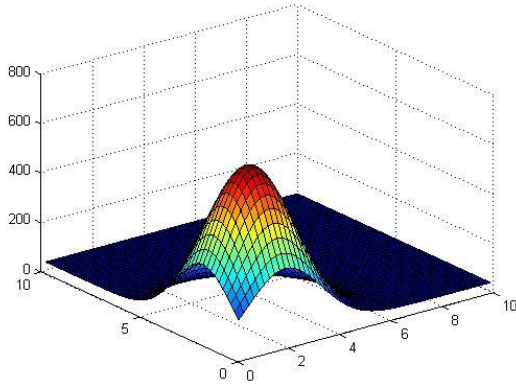
The combined boarding and alighting demand is obtained for each neighborhood by inserting (14) into (1). The resulting distributions are illustrated for each case in Fig 4.4. The distributions of boarding are identical to those of alighting for cases I – III, while for case IV the boarding and alighting distributions are distinct. In each case, the neighborhoods' highest and lowest demand densities are different by a factor of about 10, which is typical of metropolitan areas, at least in the U.S. (e.g., see <https://www.census.gov/>).



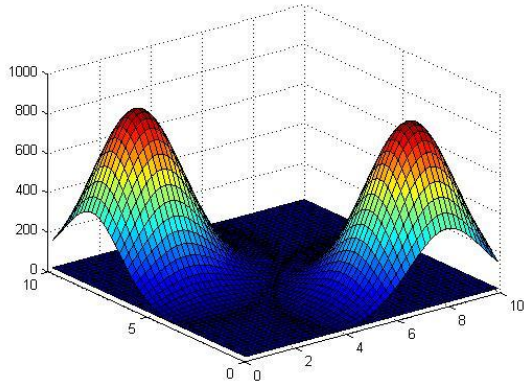
(a) Case I



(b) Case II



(c) Case III



(d) Case IV

Figure 4.4 Marginal distributions of trip origins and destinations for the four cases.

The optimal design for each case was obtained using the algorithm in Section 2.6. The parameter values used for these cases are presented in Table 4.2.¹¹ Total computation time (including evaluation) for each case was only a few seconds.

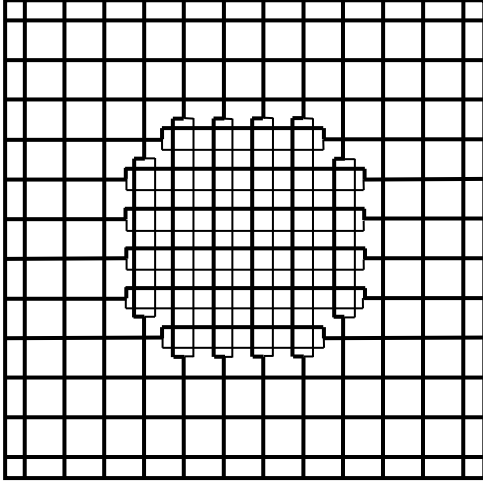
¹¹ Most of the values in Table 2 were taken from the literature (e.g. Daganzo, 2010a), though a few reflect the authors' best guesses.

Table 4.2 Input parameters of the model.

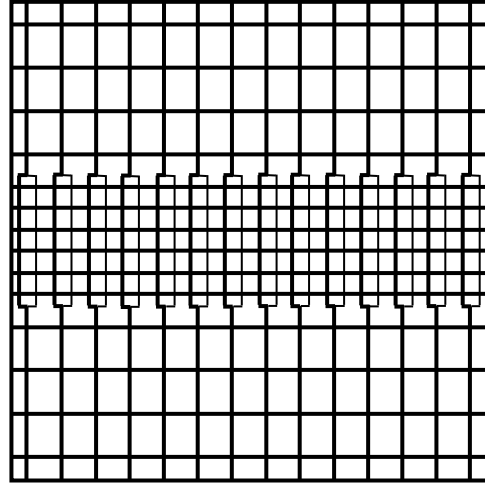
Parameter	v (km/hr)	v_w (km/hr)	τ (s)	c_s (\$/bus- stop)	c_V (\$/bus- km)	c_M (\$/bus- hr)	L (km)	W (km)	μ (\$/hour)	θ (min)
Value	25	2	30	0.3	2	40	10	10	15	1

Fig 4.5 illustrates the resulting hybrid route configuration for each case. Note that in all four cases, local routes are parts of the network configuration and are generally deployed in areas with high demand density. Of further interest, we find that although neighborhood demand densities vary by an order of magnitude, the networks' route spacings never vary by more than a factor of two. This is probably due to the EOQ¹²-type trade-offs between agency and user costs: near optimality, generalized cost is known to be insensitive to values of the decision variables (e.g. Daganzo, 1992).

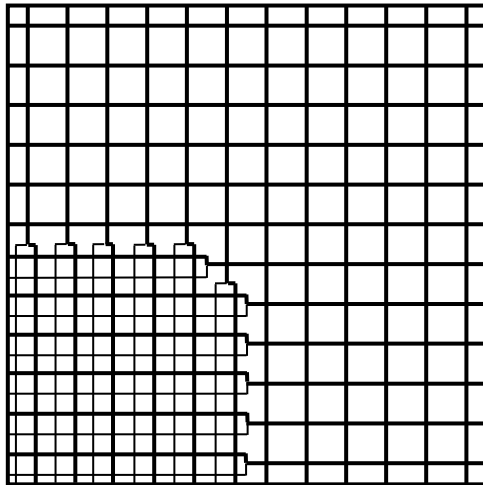
¹² Economic order quantity



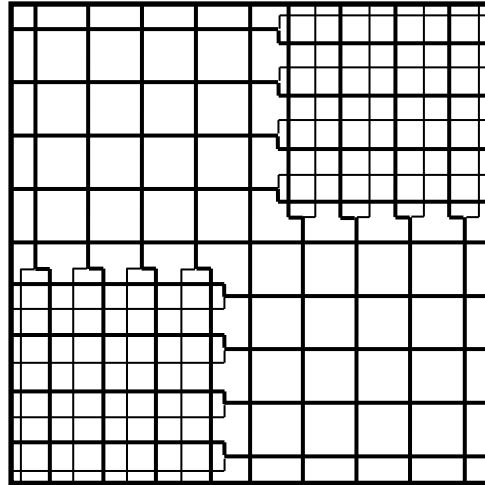
(a) Case I



(b) Case II



(c) Case III



(d) Case IV

Figure 4.5 Transit network design at convergence (drawn to scale).

We next explore for each case key aspects of our heterogeneous network design and the resulting performance; see Table 4.3. For each case, we compare these outputs against those produced by two other optimal network designs. The first of these other networks are each

designated “Network A” in Table 4.3. These were designed using our model, but under the artificial assumption that the city has a spatially homogeneous demand density throughout. Thus for each case, the demand density was in effect spread evenly over the city and the average density was used as input to the design process. Consequently, the optimal network was always a grid of main routes (only) and route spacings, w_0 and l_0 , were fixed over the entire city. These would be the kinds of networks that would be obtained via the use of conventional continuum approximation methods. The performance of each “Network A” was thereafter analyzed under its associated spatially-heterogeneous demand distribution; see again Fig. 4. These outcomes are viewed as benchmarks. On a 3.4 GHz PC, the results can be obtained between 200 and 500 CPU seconds for different cases.

A second set of networks, each designated as “Network B” in the table. These were obtained using our model while recognizing the spatially-heterogeneous demand densities, but while not allowing the inclusion of local routes. As a result, each network is once again a grid of main routes with w_0 and l_0 fixed over the city. Networks labeled “C” in Table 4.3 are those designed by our model and where local routes are used to accommodate the spatially-heterogeneous demands.

Table 4.3 shows that, for each of our four cases, the two homogeneous networks (“A” and “B”) have similar spacings, w_0 and l_0 . The similarities continue when one compares these decision variables for Networks A and B across our four cases, I – IV. This suggests that homogeneous network designs are not sensitive to spatially-heterogeneous patterns in demand. For each of the four cases, the optimal homogeneous network for heterogeneous demand (Network B) produced only modest savings in generalized costs as compared to the network

designed for an artificially homogeneous demand (Network A). And we see that user costs actually increase for Network B.

Only when local bus routes are introduced to form a heterogeneous network (Network C) do we see marked changes in w_0 and l_0 and marked reductions in costs. As regards the latter, we note for example that agency costs for our heterogeneous networks dropped by roughly 13%~26% as compared to our benchmark cases of Network A. These savings came, in part, because the heterogeneous networks can be made sparse in low-demand neighborhoods.

Adding to this, the heterogeneous configurations always produced lower user costs than did their homogeneous counterparts. Notice that this occurs despite the higher number of transfers required of the heterogeneous networks on average. This underscores the value of running closely-spaced local routes in high-demand neighborhoods. Note that a heterogeneous network diminishes a passenger's total travel time, which includes access and egress times, by as much as 7.9%; see case II in Table 4.3. Overall, the total generalized costs are lower for heterogeneous layouts than for homogeneous ones. Reductions range from 4.1% to 9.0%.

4.4. Conclusions

A continuum approximation (CA) framework for designing bus networks under spatially-heterogeneous demand was presented. A power-of-two scheme was borrowed from the field of inventory management so that main routes can be seamlessly aligned with closely-spaced local ones. Closed-form approximate expressions were formulated for the user and agency costs, and a localized optimization algorithm was developed to obtain city-wide designs.

Numerical experiments performed for select distributions of travel demand showed that the heterogeneous network configurations produce lower costs to the bus agency and to the users than do homogeneous grids. The finding underscores the value of our iterative solution method relative to conventional CA models. Recall that the use of these former models to design transit networks requires one to assume that travel demand is spatially homogeneous. And though mathematical programming approaches to transit-system design avoid the need for such an assumption, models of this latter type come with notable limitations of a different sort.

Admittedly, our proposed network configurations come with limitations of their own. Our integration of main and local routes means that passengers are required, on average, to perform greater numbers of transfers. This is a concern because transfers are onerous. In recognition of this, future work will explore how bus schedules can be coordinated using our CA framework so that passengers wait less times at transfer stops. We further intend to examine problems in which demand for transit travel varies over time as well as over space. Flexible routing designs that permit local pick-ups and drop-offs similar to those in Nourbakhsh and Ouyang (2012) may be explored for our brand of heterogeneous networks as well.

Table 4.3: Design Characteristics and Performance.

	Case (I): Mono-centric city			Case (II): Twin city			Case (III): Asym. sprawled city			Case (IV): Commuter city		
	Network A	Network B	Network C	Network A	Network B	Network C	Network A	Network B	Network C	Network A	Network B	Network C
w_0, l_0 (km)	0.50,0.50	0.50,0.50	0.77,0.77	0.50,0.50	0.50,0.50	0.71,0.91	0.50,0.50	0.50,0.50	0.77,0.77	0.50,0.50	0.59,0.59	1.10,1.10
H (min)	9	10	9	9	10	8	9	10	8	9	9	7
G	213	192	162	213	192	168	213	192	166	213	154	121
V	711	640	585	711	640	636	711	640	631	711	604	574
M	806	725	648	806	725	696	806	725	689	806	655	594
Agency cost (passenger-hr)	1730	1557	1396	1730	1557	1500	1730	1557	1486	1730	1413	1289
Agency cost improvement	-	10.0%	19.3%	-	10.0%	13.3%	-	10.0%	14.1%	-	18.3%	25.5%
A	2514	2514	2522	2517	2517	2393	2499	2499	2508	2497	2940	3116
W	1395	1550	1540	1353	1503	1207	1382	1535	1367	1485	1485	1629
IVTT	2481	2482	2235	2383	2383	2151	2790	2790	2516	5818	5571	4876
Q	144	144	174	134	134	136	142	142	177	163	163	299
User cost (passenger-hr)	6534	6687	6473	6389	6539	5885	6811	6965	6568	9962	10158	9921
User cost improvement	-	-2.4%	0.9%	-	-2.4%	7.9%	-	-2.3%	3.6%	-	-2.0%	0.4%
Total cost (passenger-hr)	8264	8244	7869	8119	8096	7385	8541	8522	8054	11692	11571	11210
Total cost improvement	-	0.2%	4.8%	-	0.3%	9.0%	-	0.2%	5.7%	-	1.0%	4.1%
Cost per passenger (hr)	0.83	0.82	0.79	0.81	0.81	0.74	0.85	0.85	0.81	1.17	1.16	1.12
Avg # of transfer (directional/spacing)	0.86/0.00	0.86/0.00	0.79/0.26	0.81/0.00	0.81/0.00	0.68/0.13	0.85/0.00	0.85/0.00	0.78/0.28	0.98/0.00	0.98/0.00	0.96/0.84
Avg passenger travel time (min)	40	40	39	39	39	35	42	42	40	61	61	60

Note: Network A: Homogeneous grid network designed under "wrong" uniform demand; Network B: Homogeneous grid network from the proposed model under correct demand; Network C: Heterogeneous grid network from the proposed model under correct demand.

Chapter 5

Bus Network Design under Spatially Heterogeneous Demand Using Modified Grid Network

5.1. Introduction

In this chapter, we present a continuum approximation method which analyzes the costs in every neighborhood. In a real world case, it is more likely that the demand varies in different locations. Therefore, a method which can analyze the cost components locally and then design transit system locally is desirable.

The main contributions of this chapter are applications of the modified grid network layout and continuum approximation method. The variable bus spacing design gives design flexibility compared to the traditional grid system to respond to demand. Therefore we can accommodate heterogeneous demand in the service region.

This chapter is organized as follows. The concept of the variable spacing transit design using continuum approximation will be explained in the following section. Section 3 analyzes and optimizes the model. Section 4 presents the numerical results, and finally Section 5 summarizes the chapter and discusses about future research directions.

5.2. Variable spacing transit design

Let us consider a service region as rectangular city with length L in E-W direction and width W in N-S direction as shown in Figure 5.1. We assume the travel demand density function $\delta(x_1, y_1, x_2, y_2)$ shows the demand from a certain neighborhood coordinate (x_1, y_1) to another neighborhood coordinate (x_2, y_2) . We assume this function varies smoothly over space.

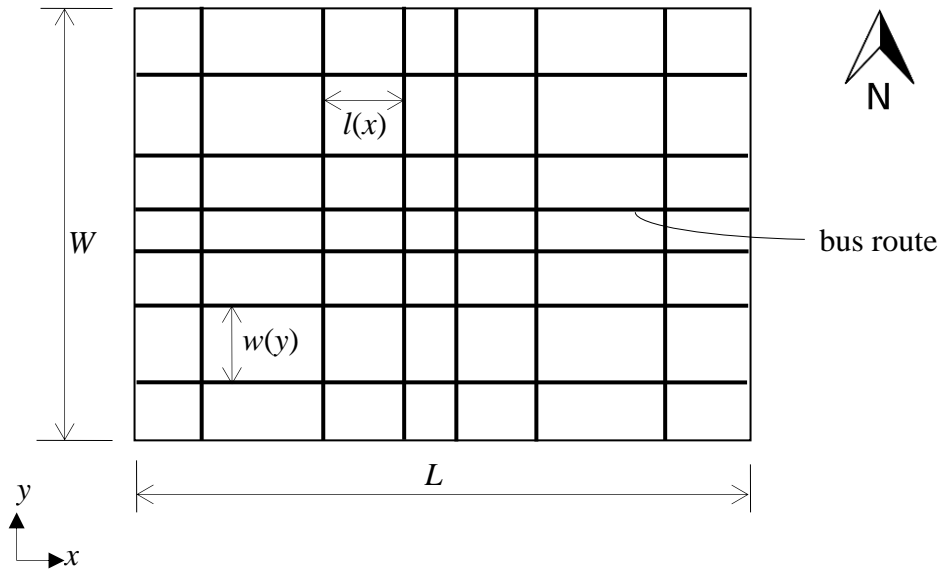


Figure 5.1 General scheme and network layout of a variable bus spacing transit network.

As shown in Figure 5.1, the general transit layout of the system is considered as a grid system. However, unlike the traditional grid system, the vertical and horizontal bus spacings are assumed to be locally dependent. Let us assume $l(x)$ and $w(y)$ are the spacings of the buses in the x and y axis (Figure 5.1). Therefore, the bus spacings in the N-S direction is a function of x and

in the E-W direction is a function of y . For simplicity, we assume constant headway, H , for all the buses.

We have made some simplifications in the previous chapter while passengers' behavior is assumed realistically. Again, passengers walk from their origins to their nearest stops and from the nearest stops to their destinations. The walking speed is assumed to be constant for passengers, v_w . Passengers travel from origin to destination by traversing the bus network via the shortest path with least amount of transfers. Passengers are assumed to choose the direction of travel randomly. The buses are assumed to have the constant cruising speed v , fixed delay per stop τ , and capacity Q .

5.3. Analysis

In this section, we analyze the agency and user costs and propose the optimum network design. The demand distribution and passenger flux are evaluated to compute the cost components.

Let $D_{\text{start}}(x, y)$ and $D_{\text{end}}(x, y)$, respectively, denote the expected travel demand rate of passengers per area per time that start and end their trip in the neighborhood (x, y) . Equation (1) shows the derivation of the expected travel demand rate from the demand function.

$$D_{\text{start}}(x, y) = \int_{\bar{y}=0}^W \int_{\bar{x}=0}^L \delta(x, y, \bar{x}, \bar{y}) d\bar{x} d\bar{y} ,. \quad (1.a)$$

$$D_{\text{end}}(x, y) = \int_{\bar{y}=0}^W \int_{\bar{x}=0}^L \delta(\bar{x}, \bar{y}, x, y) d\bar{x} d\bar{y} \quad (1.b)$$

Let $D_{\text{transf}}(x, y)$ describes the expected transfer rate of passengers per unit area per unit time in the neighborhood (x, y) . Equation (2) shows the expected number of transfer:

$$D_{\text{d_transf}}(x, y) = \frac{1}{2} \int_{\bar{y}=0}^W \int_{\bar{x}=0}^L [\delta(\bar{x}, y, x, \bar{y}) + \delta(x, \bar{y}, \bar{x}, y)] d\bar{x} d\bar{y}, \quad (2)$$

where the coefficient $\frac{1}{2}$ shows the passengers' choice of initial trip direction. The first part of the integral shows the transfer from horizontal to vertical buses and the second part shows the transfer from vertical to horizontal buses at any point (x, y) .

We can also compute the flux for all 4 bus directions using the assumptions mentioned in previous section. The following equation shows the passenger flux for the eastbound bus.

$$F_E(x, y) = \frac{1}{2} \int_{x_1=0}^x \left[\int_{x_2=x}^L \int_{y_2=0}^W \delta(x_1, y, x_2, y_2) dy_2 dx_2 \right] dx_1 + \frac{1}{2} \int_{x_2=x}^L \left[\int_{x_1=0}^x \int_{y_1=0}^W \delta(x_1, y_1, x_2, y) dy_1 dx_1 \right] dx_2. \quad (3)$$

In the above equation, the coefficient of $\frac{1}{2}$ shows the passengers' initial choice of direction. Similarly, $F_W(x, y)$, $F_S(x, y)$ and $F_N(x, y)$ denote the flux for the westbound, southbound and northbound, respectively. To find the northbound direction flux, swap the x and y and L and W , respectively. Let us $Q_E^{\max}(y)$ be the maximum flux for the eastbound direction (equation (4)). Using the same concept we can obtain the maximum flux in other directions, $Q_W^{\max}(y)$, $Q_S^{\max}(y)$ and $Q_N^{\max}(y)$.

$$Q_E^{\max}(y) = \max_{x \in [0, L]} \{F_E(x, y)\}. \quad (4)$$

5.3.1. Agency Costs

We consider three agency cost components: bus stopping to load and unload passengers, G ; the total vehicular distance traveled, V ; and the total vehicular time expended, M . Equations (5)-(7) show the cost components in neighborhood (x,y) . The unit of the cost is \$ per unit area per unit time.

$$G(x, y) = 4c_s l^{-1}(x)w^{-1}(y)H^{-1}, \quad (5)$$

$$V(x, y) = 2c_v H^{-1} \left[l^{-1}(x) + w^{-1}(y) \right], \quad (6)$$

$$M(x, y) \approx \frac{c_M}{c_v} V(x, y) \left[v^{-1} + \frac{2\tau}{l(x) + w(y)} \right], \quad (7)$$

In the equations (5)-(7), the coefficients c_s , c_v , and c_M are the unit operation costs per bus stop, per bus mile and per bus hour, respectively. Equation (5) determines the cost due to stopping, where $4l^{-1}(x)w^{-1}(y)H^{-1}$ is the number of stops. Equation (6) represents the total travel distance for all buses per unit area, $2[l^{-1}(x) + w^{-1}(y)]$, per unit time, H^{-1} . Equation (7) calculates the total vehicular time by using Equation (6) and adjusting the coefficients and multiplying by commercial speed of the buses, $v \left[1 + \frac{2\tau v}{l(x) + w(y)} \right]^{-1}$.

5.3.2. User Costs

Four cost components were considered in user cost: A , the time passenger spend to walk from their origin to the nearest stop and from their last stop to their destination; W , the passengers waiting time both at their first stop and transfer location; $IVTT$, the time passengers spend in a bus; and Q , the penalty associated with each transfer. Equations (8) - (11) calculate the user costs.

$$A(x, y) = \frac{1}{4v_w} [l(x) + w(y)] [D_{\text{start}}(x, y) + D_{\text{end}}(x, y)] \quad (8)$$

$$W(x, y) \approx \frac{H}{2} [D_{\text{start}}(x, y) + D_{\text{d_transf}}(x, y)], \quad (9)$$

$$IVTT(x, y) = \frac{1}{v} (F_W(x, y) + F_E(x, y) + F_S(x, y) + F_N(x, y)) + \tau \left(\frac{F_E(x, y) + F_W(x, y)}{w(y)} + \frac{F_N(x, y) + F_S(x, y)}{l(x)} \right) \quad (10)$$

$$Q(x, y) = \theta [D_{\text{transf}}(x, y)] \quad (11)$$

where θ in equation (11) is the penalty associated with each transfer. Equation (8) calculate the users' walking time based on the travel distance, boarding and alighting demand, and passengers' walking speed. Equation (9) obtains the passengers' waiting time assuming uniform arrival and no synchronization. Equation (10) underscores the in vehicle travel time in which the first term is the time that passengers spend while the bus is cruising, and the second term is the time that the passengers spend while the bus is loading and unloading other passengers.

5.3.3. Optimum Network Design

Knowing the agency and user cost, we want to find the optimum network design. We use the average passenger time value, μ , to convert the agency cost unit, \$, to an equivalent user cost, time. Equation (12.a) minimizes the total cost including the time value of agency cost and user cost over the service region. The decision variables are H , $l(x)$ and $w(y)$ for every neighborhood (x, y) . Equation (12.b) implements the non-negativity constraint for the decisions variables.

$$\text{Min } \int_{y=0}^W \int_{x=0}^L Z(x, y) dx dy \quad (12.a)$$

$$\text{s.t. } l(x), w(y), H \geq 0 \quad (12.b)$$

where $Z(x, y) = \frac{1}{\mu} [G(x, y) + V(x, y) + M(x, y)] + [A(x, y) + W(x, y) + IVTT(x, y) + Q(x, y)]$ is the local generalized cost per area per time at neighborhood (x, y) .

The objective function is a convex function of the decision variables, H , $l(x)$ and $w(y)$. For solving this problem different methods can be used. We can apply numerical search to find the optimum value of the decision variables.

5.4. Numerical results

In this section we apply the proposed model to some hypothetical cities and evaluate the model performance. Four different demand functions were examined similar to the previous chapter with slight changes to their density functions. The general demand density of the four cases is shown in equation (14):

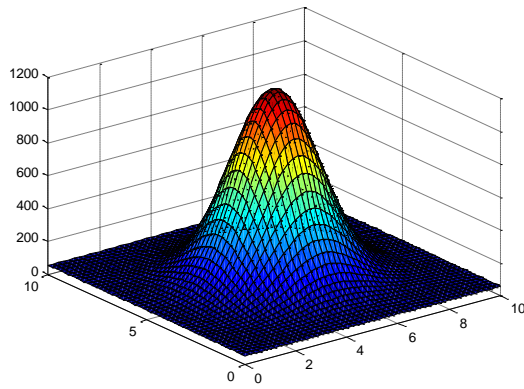
$$\delta(x_1, y_1, x_2, y_2) = \prod_{i=1}^2 \left(a_1 + a_2 \exp \left[-(a_3 x_i - b_i)^2 - (a_4 y_i - \bar{b}_i)^2 \right] \right), \quad (14)$$

where the parameters are specified in Table 5.1 for all cases. Similar to previous chapter, the parameters are set such that the total demand in the city in all four cases is 10,000 passengers per hour for the entire city.

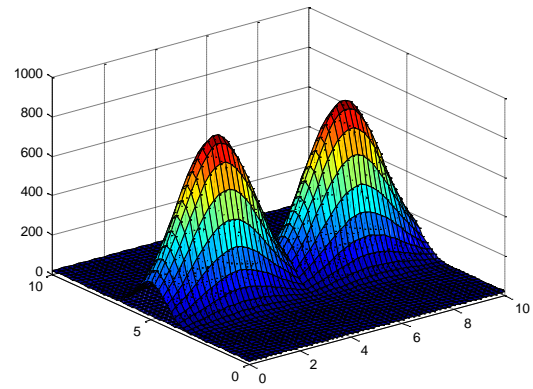
Table 5.1 Demand distribution parameters in cases (I)-(IV).

Case	Parameters							
	a_1	a_2	a_3	a_4	b_1	b_2	\bar{b}_1	\bar{b}_2
(1): Mono-centric city	0.0016	0.065	0.5	0.5	2.5	2.5	2.5	2.5
(2): Twin city	0.0034	0.137	0.5	1	1.25	3.75	5	5
(3): Asymmetrically sprawled city	0.0021	0.081	0.5	0.5	1	1	1	1
(4): Commuter city	0.00044	0.035	0.5	0.5	1	4	4	1

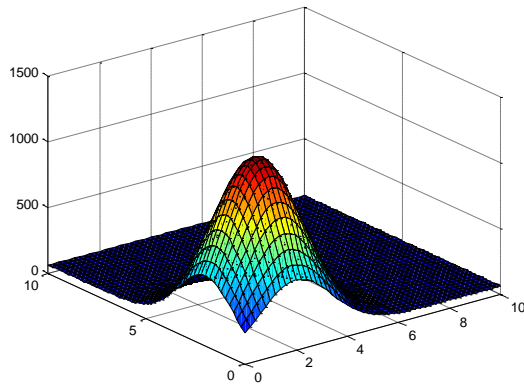
Figure 5.2 shows the trip production, $D_{\text{start}}(x, y)$, and trip attraction, $D_{\text{end}}(x, y)$, distribution function for all 4 cases. Cases I and III have the same boarding and alighting graph while cases II and IV have different ones which both of them are shown in same graph.



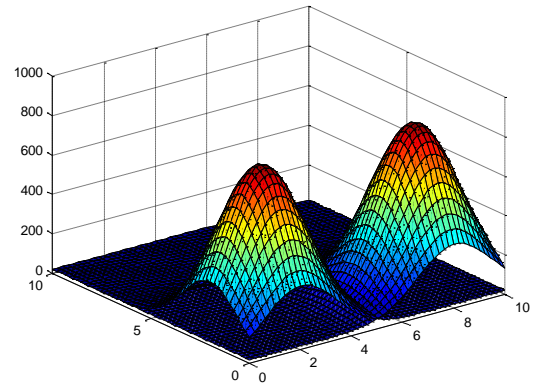
(a) Case I



(b) Case II



(c) Case III



(d) Case IV

Figure 5.2 Marginal distributions of trip origins and destinations for the four cases.

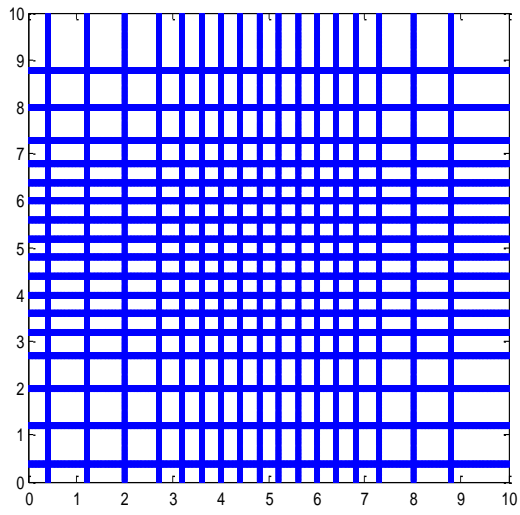
We applied the optimum network design for all 4 cases. The parameters in Table 5.2 were used as input data. Most input data were adapted from literature (Daganzo, 2010a and Estrada et al, 2011). A numerical search method was used to find the optimum network design. The running

time for the different cases took between 100 to 300 CPU seconds on a 3.4 GHz desktop with 4GB memory.

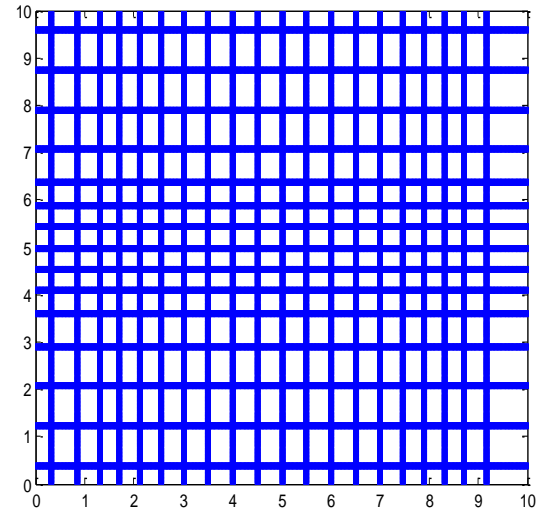
Table 5.2 Input parameters of the model.

Parameter	v (km/hr)	v_w (km/hr)	τ (s)	c_s (\$/bus- stop)	c_V (\$/bus- km)	c_M (\$/bus- hr)	L (km)	W (km)	μ (\$/hour)	θ (min)
Value	25	2	30	0.3	2	40	10	10	15	1

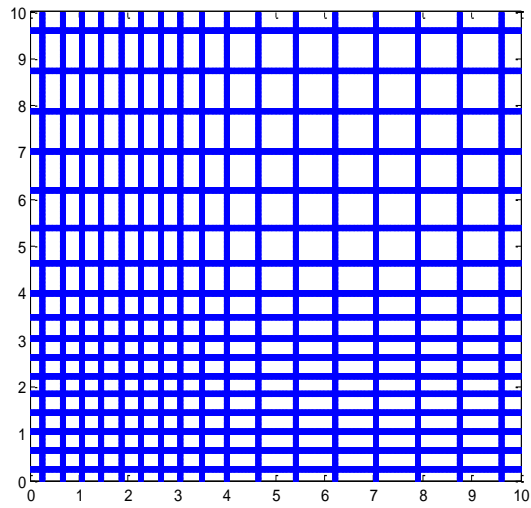
Figure 5.3 illustrates the optimal bus route configuration for each case. The optimal bus routes, thick blue lines, tend to be relatively denser in higher demand areas and sparser in lower demand areas.



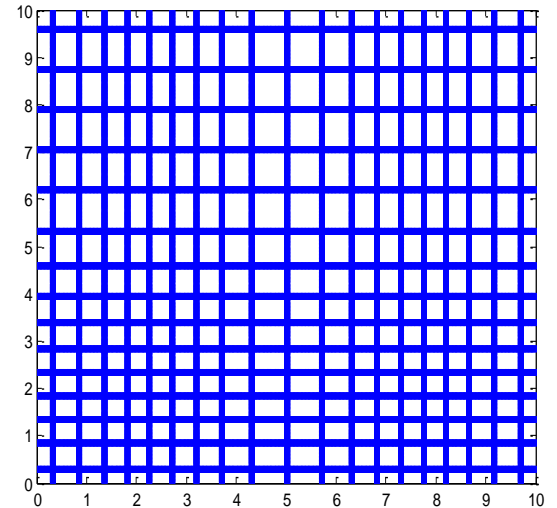
(a) Case I



(b) Case II



(c) Case III



(d) Case IV

Figure 5.3 Transit network design at convergence (drawn to scale).

All the cost components including agency and user costs are shown in Table 5.3. As it is illustrated, the proposed network design improves the cost 3% to 6% compared to both Network A, homogenous demand assumption with homogeneous design, and network B, heterogeneous demand assumption with homogeneous design. In Case I, mono centric demand distribution, the bus spacings narrow in central and expand in the peripheral part. Having higher demand in the central area, the average passengers walking time decreases. In addition, the average bus travel distance decreases in the new design. As a result both user and agency costs drop. Case II, twin city, shows improvement by reducing the vertical spacings along the both city centers. Case III has the highest cost saving by reducing the spacings in both directions along the corner and increasing them on the other areas. In the last case, commuter city, improvement is minimal because the peak trip production and attraction are at the opposite corner. However in this case the total cost improves 3% despite of the 1% growth in user cost.

The passenger number of transfers is between 0.83 and 0.89 that shows much less variation compared to design with local buses. That is because all the transfers are directional ones and there is no spacing transfer. Therefore the final design and cost components associated with it are not sensitive to the transfer discomfort, θ .

Table 5.3 Cost components for all cases.

	Case I			Case II			Case III			Case IV		
	Network A	Network B	Network C	Network A	Network B	Network C	Network A	Network B	Network C	Network A	Network B	Network C
<i>H</i> (min)	9'	10'	9'	9'	10'	9'	9'	10'	9'	9'	10'	9'
Agency costs (time/hour)	1730	1557	1516	1730	1557	1430	1730	1557	1391	1730	1557	1408
G	213	192	172	213	192	160	213	192	154	213	192	152
V	711	640	640	711	640	607	711	640	592	711	640	604
M	806	725	704	806	725	663	806	725	645	806	725	652
User costs (time/hour)	6494	6648	6355	7267	7421	6688	6782	6935	6444	7938	8095	7946
A	2499	2499	2443	2496	2496	2416	2488	2488	2452	2484	2484	2638
W	1386	1540	1390	1384	1538	1283	1375	1528	1346	1409	1566	1398
IVTT	2467	2467	2378	3246	3246	2857	2778	2778	2507	3898	3898	3764
Q	142	142	144	141	141	132	141	141	139	147	147	146
Total costs (time/hour)	8224	8205	7871	8997	8978	8118	8512	8492	7835	9668	9652	9354
Total cost per observed passenger	0.83	0.83	0.79	0.90	0.90	0.85	0.86	0.86	0.81	0.97	0.97	0.94
Total cost Improvement	4%			5%			6%			3%		
Total cost Improvement compare to bench mark	4%			6%			6%			3%		
Passenger # of transfers	0.86	0.86	0.87	0.85	0.85	0.83	0.86	0.86	0.86	0.89	0.89	0.88
Passenger travel time (min)	39'	40'	38'	44'	45'	40'	41'	42'	39'	48'	49'	48'
Agency cost improvement	13%			17%			22%			22%		
Passenger travel time improvement	1%			5%			3%			-1%		
Total Demand	9,999	9,999	9,999	9,978	9,978	9,978	9937	9937	9,999	9915	9915	9915
Observed Demand	9,937	9,937	9,870	9,972	9,972	9,502	9893	9893	9,675	9951	9951	9901

5.5. Conclusions

This chapter proposes a continuum approximation framework for bus network design. A modified grid system as a general layout of the bus network is also proposed for accommodation of the demand variation over space. The modified grid network gives flexibility to the bus lines to adjust their spacing based on the demand. Using the CA method, we obtain closed-form formula for all cost components including agency and user cost. The optimization techniques are applied to get the optimum transit network layout as well as buses headway.

We applied our model for some hypothetical cases considering different demand distributions. The results show that for these heterogeneous demand distributions, the modified grid network can reduce the total cost effectively compared to the traditional grid system.

Compared to the transit network design with local buses proposed in previous chapter, the proposed method has some advantages and disadvantages. The simplified grid network incurs less transfers and easier to implement for real cases. However, it works well for some demand distribution patterns. For example if there is a high demand in a local area, the bus spacings in along the horizontal and vertical directions reduce and incur extra agency cost. On the other hand, the local grid idea can address this issue very well while it might cause extra transfers for a portion of passengers.

For future research we can consider extending the ideas of either modified grid layout or CA method to flexible-route transit network. In addition, we can try to design transit systems for more general service region shapes as well as considering different transit system layout. As an

extension of heterogeneous demand over space, considering demand variation over time (variable headways) would be another possible extension.

Chapter 6

Continuum Approximation Method for Bus Network Design for Polar System with Heterogeneous Demand

6.1. Introduction

As mentioned earlier in Chapter 2, most literature on transit network design assumes the shape of the city to be rectangular or square. However, lots of cities in the real world has circular or pie shapes with a city core (e.g., Paris and Beijing). In addition, the street structure of some cities does not necessarily form an EW-NS grid. In this chapter, we aim to extend the analysis method that we used in previous chapters to circular shape service regions and design transit system with a polar layout. Similar to the previous two chapters (Chapters 4 and 5) we use a continuum approximation (CA) method to analyze the cost components and design the optimum transit system. The CA method enables us to get closed-form formulas for all cost components even under a heterogeneous passenger O/D demand distribution. We further apply the proposed method to some hypothetical cases in order to give urban planners insights on polar network design.

The organization of this chapter is as follows. Section 6.2 describes concepts and assumptions of the polar transit network. Section 6.3 analyzes the system cost and optimum network design. Section 6.4 applies the design techniques and presents a system analysis for some hypothetical cases, and Section 6.5 concludes the chapter and discusses about future research directions.

6.2. Concept

This section describes the network characteristics and concepts. The service region is assumed to be a circular area with radius R . We assume the buses travel either radially toward the center or toward the boundary, or circularly clockwise or counterclockwise. The bus stops are located at the intersection of radial and circular bus lines. Network streets are assumed to be dense enough to accommodate the radial and circular bus lines everywhere. Both circular and radial buses are assumed to travel at cruising speed v . The boarding and alighting time is τ units of time per stop.

Passengers are assumed to choose the closest bus stops from their origins or destinations. As before, we assume passengers choose the shortest bus network path for travelling from their origin (r_1, θ_1) to their destination (r_2, θ_2) . Figure 6.1 shows the travel path of passengers for two cases based on the degree difference between origin and destination. If this difference is less than 2 radian the passengers travel via the radial and shortest circular bus lines, as shown in Figure 6.1.a; otherwise they travel using two radial buses that pass their origin and destination, as shown in Figure 6.1.b. In both cases, passengers transfer at most once, therefore the average number of transfers of the system is always less than or equal to 1. The transfers can happen either at an intersection of circular and radial bus routes (Figure 6.1.a) or at the center of the city (Figure 6.1.b). In the cost analysis we consider the discomfort associated with transfers as a virtual cost where we assume $disc$ unit time per stop.

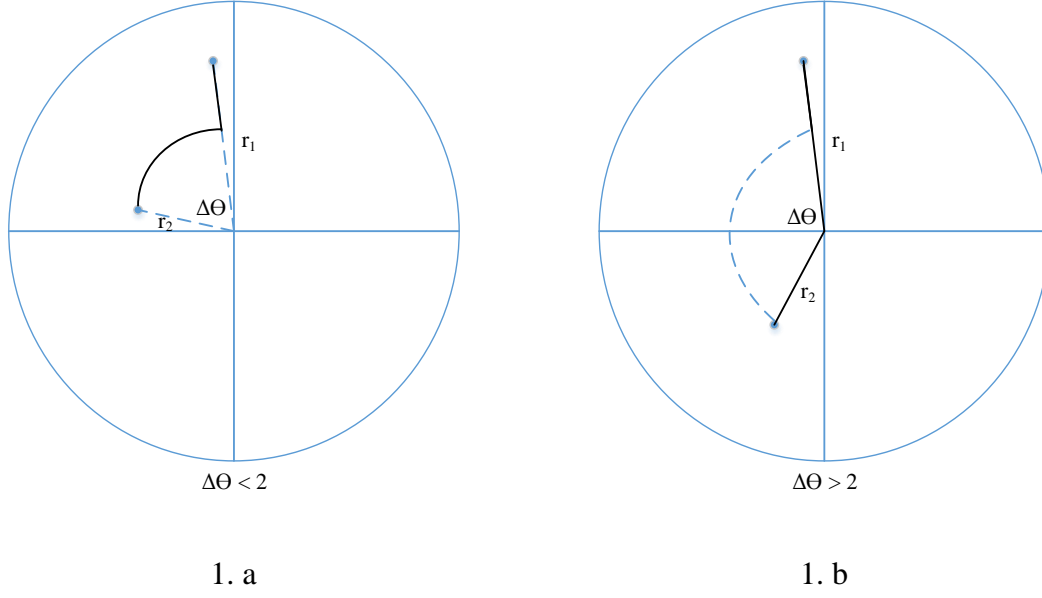


Figure 6.1 Passengers travel path for a. $\Delta\theta < 2$ and b. $\Delta\theta > 2$.

We assume the demand density is a smooth function over time and space $\delta(r_1, \theta_1, r_2, \theta_2)$ which represents the demand from the origin neighborhood (r_1, θ_1) to the destination neighborhood (r_2, θ_2) .

6.3. System analysis

Before we start the analysis of the cost components, let us examine the demand at any origin, destination or transfer point (r, θ) . We let $D_{\text{start}}(r, \theta)$ and $D_{\text{end}}(r, \theta)$ represent the expected travel demand rates of passengers per area per time that start and end their trip in neighborhood (r, θ) respectively. Equation (1) shows the calculation.

$$D_{\text{start}}(r, \theta) = \int_{\bar{\theta}=0}^{2\pi} \int_{\bar{r}=0}^R \delta(r, \theta, \bar{r}, \bar{\theta}) \bar{r} d\bar{r} d\bar{\theta} , \quad (1.a)$$

$$D_{\text{end}}(r, \theta) = \int_{\bar{\theta}=0}^{2\pi} \int_{\bar{r}=0}^R \delta(\bar{r}, \bar{\theta}, r, \theta) \bar{r} d\bar{r} d\bar{\theta} . \quad (1.b)$$

$D_{\text{transf}}(r, \theta)$ represents the expected number of passengers per area per time that transfer in neighborhood (r, θ) from a radial to a ring line or vice versa. The derivation is shown in Equation (2).

$$D_{\text{transf}}(r, \theta) = \int_{\bar{\theta}=\theta-2}^{\theta+2} \int_{\bar{r}=r}^R \delta(\bar{r}, \theta, r, \bar{\theta}) \bar{r} d\bar{r} d\bar{\theta} + \int_{\bar{\theta}=\theta-2}^{\theta+2} \int_{\bar{r}=r}^R \delta(r, \bar{\theta}, \bar{r}, \theta) \bar{r} d\bar{r} d\bar{\theta} \quad r \neq 0$$

$$D_{\text{transf}}(0, \theta) = \int_{r=0}^R \int_{\bar{\theta}=\theta+2}^{\theta+2\pi-2} \int_{\bar{r}=0}^R \delta(r, \theta, \bar{r}, \bar{\theta}) \bar{r} d\bar{r} d\bar{\theta} r dr \quad r = 0 \quad , \quad (2)$$

Figure 6.2 shows how Equation (2) calculates the transfer. The transfers that happen in the neighborhood (r, θ) , where $r \neq 0$, is generated by the passengers who want to travel from/to the shaded area in the circular bus line to/from the radial bus line. The transfer at the city center is made by passengers who only ride on radial bus lines for their travel.

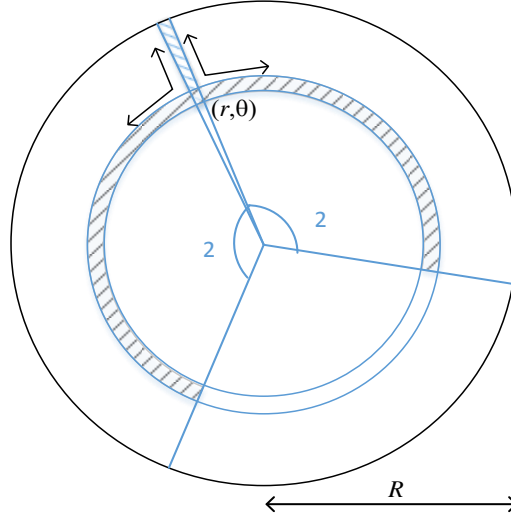


Figure 6.2 Transfer in the neighborhood of (r, θ) .

In addition to demand analysis, we also need to know the passenger flux at any neighborhood (r, θ) . Let $F_{CW}(r, \theta)$ and $F_{CC}(r, \theta)$ denote the flux of circular buses in clockwise and counterclockwise directions, respectively. Also, let $F_I(r, \theta)$ and $F_O(r, \theta)$ represent the flux of radial buses in inward and outward directions, respectively. Figure 6.3 shows all four different fluxes at point (r, θ) .

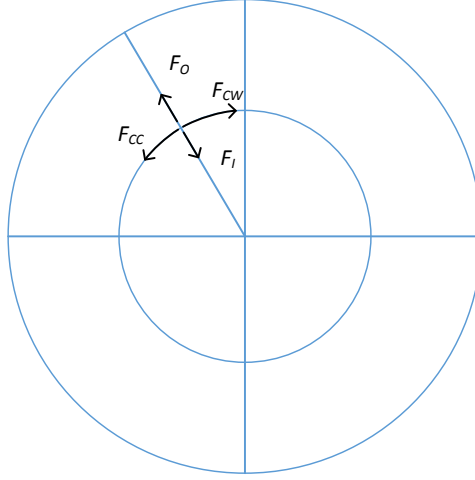


Figure 6.3 The flux representation at the neighborhood of (r, θ) .

Equation (3) denotes the flux formula for all four directions. Appendix C provides the derivation of the formulas.

$$F_{CW}(r, \theta) = \int_{\theta_1=\theta}^{\theta+2} \left[\int_{r_2=r}^R \int_{\theta_2=\theta_1-2}^{\theta_1} \delta(r, \theta_1, r_2, \theta_2) r_2 d\theta_2 dr_2 \right] d\theta_1 + \int_{\theta_2=\theta-2}^{\theta} \left[\int_{r_1=r}^R \int_{\theta_1=\theta_2}^{\theta_2+2} \delta(r_1, \theta_1, r, \theta_2) r_1 d\theta_1 dr_1 \right] d\theta_2 \quad (3.a)$$

$$F_{CC}(r, \theta) = \int_{\theta_1=\theta-2}^{\theta} \left[\int_{r_2=r}^R \int_{\theta_2=\theta_1}^{\theta_1+2} \delta(r, \theta_1, r_2, \theta_2) r_2 d\theta_2 dr_2 \right] d\theta_1 + \int_{\theta_2=\theta}^{\theta+2} \left[\int_{r_1=r}^R \int_{\theta_1=\theta_2-2}^{\theta_2} \delta(r_1, \theta_1, r, \theta_2) r_1 d\theta_1 dr_1 \right] d\theta_2 \quad (3.b)$$

$$F_I(r, \theta) = \int_{r_1=r}^R \left[\int_{r_2=0}^r \int_{\theta_2=\theta+2}^{\theta-2+2\pi} \delta(r_1, \theta, r_2, \theta_2) r_2 d\theta_2 dr_2 \right] r dr_1 + \int_{r_1=r}^R \left[\int_{r_2=0}^r \int_{\theta_2=\theta-2}^{\theta+2} \delta(r_1, \theta, r_2, \theta_2) r_2 d\theta_2 dr_2 \right] r dr_1 \quad (3.c)$$

$$F_O(r, \theta) = \int_{r_2=r}^R \left[\int_{r_1=0}^r \int_{\theta_1=\theta+2}^{\theta-2+2\pi} \delta(r_1, \theta_1, r_2, \theta) r d\theta_1 dr_1 \right] r_2 dr_2 + \int_{r_2=0}^r \left[\int_{r_1=r}^R \int_{\theta_1=\theta-2}^{\theta+2} \delta(r_1, \theta_1, r_2, \theta) r d\theta_1 dr_1 \right] r_2 dr_2 \quad (3.d)$$

For simplicity and ease of synchronization, we assume the same headway for all the buses. We define three decision variables: distance between two circular bus lines, $R(r, \theta)$, angle between two radial lines, $\Theta(r, \theta)$, and, headway for all buses, H .

6.3.1. Agency costs

We consider three agency costs. G , the bus stopping to load and unload passengers; V , the total vehicular distance and M , the total vehicular time. Equation (4)-(6) show the agency cost components.

$$G(r, \theta) = 4c_s R^{-1}(r, \theta) r^{-1} \Theta^{-1}(r, \theta) H^{-1}, \quad (4)$$

$$V(r, \theta) = 2c_v H^{-1} [R^{-1}(r, \theta) + r^{-1} \Theta^{-1}(r, \theta)], \quad (5)$$

$$M(r, \theta) \approx \frac{c_M}{c_v} V(r, \theta) \left[v^{-1} + \frac{2\tau}{R(r, \theta) + r\Theta(r, \theta)} \right]. \quad (6)$$

Equation (4) represents the bus stopping cost where c_s is the unit operation cost per stop, where 4 is number of buses passing in each stop, $R(r, \theta)r\Theta(r, \theta)$ is the area each bus stop serves, and H is the headway of the buses. In Equation (5), c_v is the unit operation cost per bus mile, $2[R^{-1}(r, \theta) + r^{-1}\Theta^{-1}(r, \theta)]$ is the bus travel distance for circular and radial buses in both directions per unit area, and H^{-1} is the unit time. When calculating the total vehicular time, we modify the previous formula by adjusting the coefficients and multiplying it by the inverse of the

commercial bus speed, $\left[v^{-1} + \frac{2\tau}{R(r, \theta) + r\Theta(r, \theta)} \right]$. Note that c_M is the unit operation cost per bus hour.

6.3.2. User costs

Four cost components are considered as user costs: A , the access time; W , the waiting time; $IVTT$, the in vehicle travel time and Q , the transfer discomfort. Equations (7)-(10) show the user cost components.

$$A(r, \theta) \approx \frac{1}{4v_w} [R(r, \theta) + r\Theta(r, \theta)] [D_{\text{start}}(r, \theta) + D_{\text{end}}(r, \theta)], \quad (7)$$

$$W(r, \theta) = \frac{H}{2} [D_{\text{start}}(r, \theta) + D_{\text{transf}}(r, \theta)], \quad (8)$$

$$IVTT(r, \theta) = \frac{1}{v} (F_I(r, \theta) + F_O(r, \theta) + F_{CW}(r, \theta) + F_{CC}(r, \theta)) + \tau \left(\frac{F_I(r, \theta) + F_O(r, \theta)}{R(r, \theta)} + \frac{F_{CW}(r, \theta) + F_{CC}(r, \theta)}{r\Theta(r, \theta)} \right), \quad (9)$$

$$Q(r, \theta) = \text{disc} [D_{\text{transf}}(r, \theta)]. \quad (10)$$

Equation (7) denotes the walking time of passengers from their origin to the nearest stop and from their last stop to their destination. In this equation, we assume the average walking distance of users is $\frac{1}{4} [D_{\text{start}}(r, \theta) + D_{\text{end}}(r, \theta)]$, the average passengers walking speed is v_w , and size of area

served is $dR(r, \theta) \times rd\Theta(r, \theta)$. In the case that the bus lines are not parallel to city streets the equation overestimates the walking distance. The next equation represents the waiting time of the passengers at their origin and transfer point. We assume no synchronization and variation in buses. Equation (9) denotes the in vehicle travel time of passengers where the first term represents the travel time while bus is travelling, and the second term indicates the time that passengers spend while the bus is loading and unloading other passengers. The last user cost component, Equation (10), is the discomfort associated with transfers.

6.3.3. Network Optimization

The network design problem is minimizing the total system cost and determining the optimum value for decision variables, $R(r, \theta)$, $\Theta(r, \theta)$, and H . Equation (11) shows the objective function and constraints:

$$\text{Min} \int_{\theta=0}^{2\pi} \int_{r=0}^R Z(r, \theta) r dr d\theta, \quad (11.a)$$

$$R(r, \theta) \geq 0, \quad \forall(r, \theta), \quad (11.b)$$

$$\Theta(r, \theta) \geq 0, \quad \forall(r, \theta), \quad (11.c)$$

$$H \geq 0. \quad (11.d)$$

Where $Z(r, \theta) = \frac{1}{\mu} [G(r, \theta) + V(r, \theta) + M(r, \theta)] + [A(r, \theta) + W(r, \theta) + IVTT(r, \theta) + Q(r, \theta)]$. In order to

unify the user and agency unit cost, we use μ in this equation to denote an average passenger

value of time. Different solution techniques, including numerical search, can be used to solve this problem.

6.4. Numerical examples

In this section we want to design a bus system for a circular city considering both homogeneous (Case I) and heterogeneous (Cases II and III) demand distributions. For all the cases the demand stays mono centric. The demand distributions for all three cases are as follows.

- Case I: $\delta(r_1, \theta_1, r_2, \theta_2) = 1.62$
- Case II: $\delta(r_1, \theta_1, r_2, \theta_2) = (5 - r_1)(5 - r_2) / 2.32$
- Case III: $\delta(r_1, \theta_1, r_2, \theta_2) = (25 - r_1)^2 (25 - r_2)^2 / 96.80$

The parameters in the demand distribution are set, such that the total demand for all cases is 10,000 passengers per hour. The parameters used as an input data in the case studies are shown in Table 6.1.

Table 6.1 Input parameters of the model.

Parameter	v (km/hr)	v_w (km/hr)	τ (s)	c_s (\$/bus- stop)	c_V (\$/bus- km)	c_M (\$/bus- hr)	R (km)	μ (\$/hour)	$disc$ (min)
Value	25	2	30	0.3	2	40	10	15	1

The optimum network design is applied to both cases. We apply the numerical search to find the optimum value of radial and circular bus lines and common bus headway. The running time on a 3.4 GHz desktop with 4GB memory is between 100 to 300 CPU seconds. Figure 6.4 shows the optimum transit network design layout for the three demand distributions.

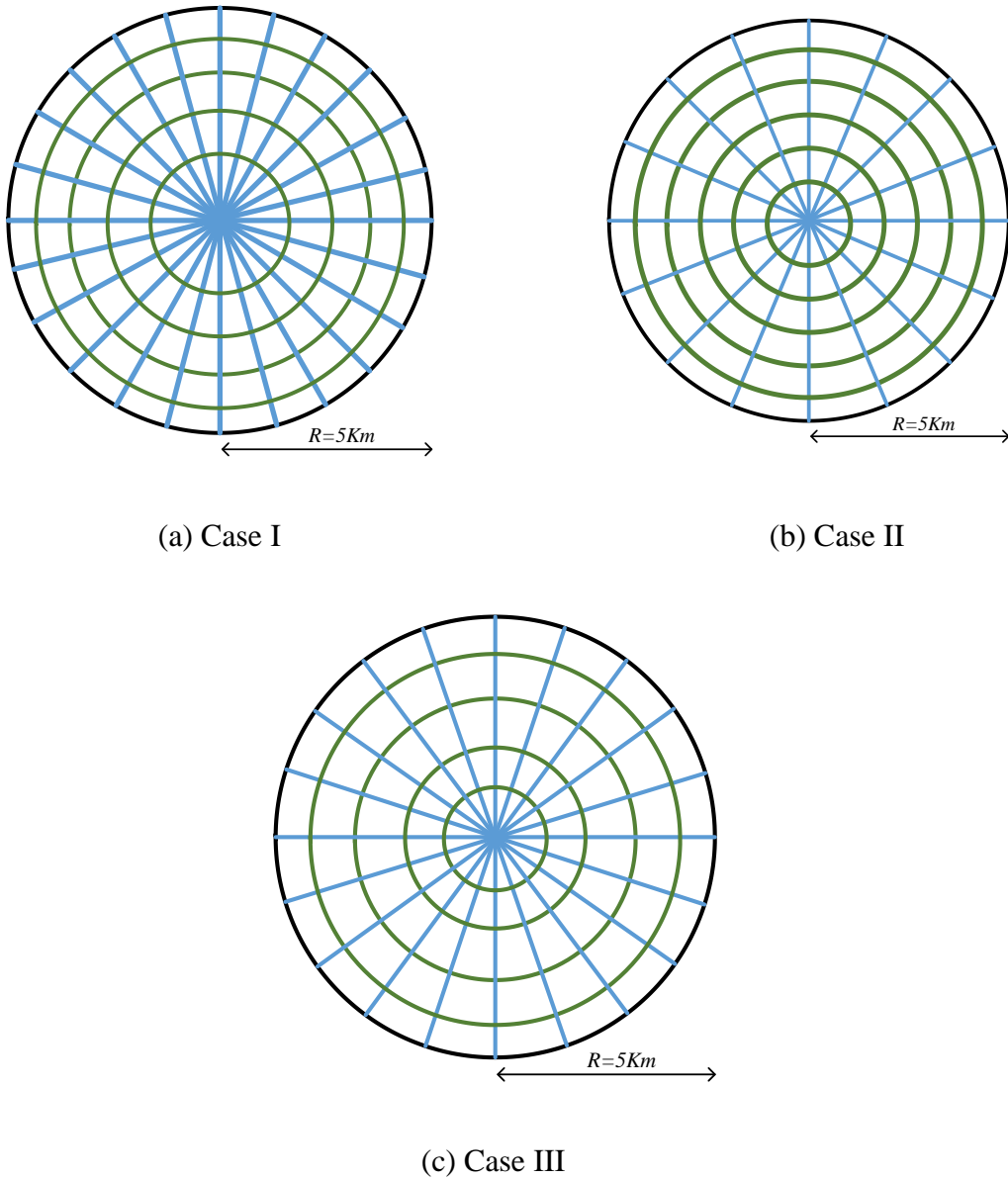


Figure 6.4 Transit network design at convergence (drawn to scale).

As it shown in Figure 6.4, the first case has the densest network compared to the other two networks both in radial and circular directions. Case II has denser circular bus lines while it has sparser radial lines. The cost components are shown in Table 6.2.

Table 6.2 Cost components parameters.

	Case I	Case II	Case III
H (min)	8	7	7
R (km)	0.20	0.32	0.30
Θ (km)	0.5-1.5	0.6-0.13	0.6-1.4
Agency costs (time/hour)	1280	1,080	1,126
G	126	96	100
V	563	496	508
M	590	488	517
User costs (time/hour)	11,997	11,667	11,883
A	3,348	3,663	3,693
W	1,337	1,167	1,166
IVTT	7,145	7,172	6,875
Q	166	166	166
Total costs (time/hour)	13,277	13,248	13,009
Total Demand	10,000	10,000	10,000

We observed the total and user costs decrease from Case I to Case III, mainly because the IVTT decreases. The first case has the highest agency cost compared to other two cases while

case II and III have almost the same cost. This statement governs on all agency cost components as well. Since we have denser transit network in case I we can observe less walking time compared to case II and III. The IVTT decreases from Case I to Case III because the demand distribution becomes denser in the center and trip length becomes shorter. The waiting time and discomfort associated to transfer remain the same for all cases.

6.5. Conclusion

In this chapter we examined a continuum approximation method to analyze the cost components of a circular city with radial and circular bus lines. Closed-form formulas are derived for both agency and user cost components. By minimizing the total system cost, we determined the optimum transit network layout. Numerical results show the implementation of the proposed design method on some hypothetical cities.

The proposed model has some limitations. For some special demand distributions variable bus spacings do not work much better than fixed bus spacings. For future study from an academic point of view, we will try to examine the pie shape polar network design. From a practical point of view, we want to apply the model to real world case and compare the results with their current system as well as other design approaches.

Chapter 7

Real World Case Study: Champaign-Urbana Safe Ride

7.1. Introduction

In the flexible-route transit system design we have proposed a transit network for a square city with homogeneous demand. In this section we generalize the shape of the service region to rectangle. By making this assumption we can analyze most cities with grid streets.

The structured flexible-route transit in Chapter 3 so far has only been applied to hypothetical numerical examples. In this chapter we extend the design framework to a real-world case for the twin city of Urbana-Champaign. In this regard, we get the required data from the Champaign-Urbana Mass Transit District (CUMTD) agency. The current safe-ride system operates manually and vehicles are dispatched based on a simple first-call first-serve rule. Passengers are picked up and dropped off in three zones of the extended University of Illinois campus area. They may transfer at three locations if needed (Illini union, Armory, and ISR station). Figure 7.1 illustrates the service region and three transfer points.

Let's consider a rectangle with horizontal and vertical sides D_x and D_y (km) as shown in Figure 7.2. We divided the city into an inner rectangle with horizontal and vertical sides d_x and d_y (km) and an outside area (peripheral). We use the ratios $\alpha_x = \frac{d_x}{D_x}$ and $\alpha_y = \frac{d_y}{D_y}$ for respectively horizontal and vertical relative size of inner part to the entire city. The transit system in N-S and E-W directions has N_y and N_x tubes respectively. Other assumptions and notations are the same as Chapter 3. Similarly the design problem is to find the optimal combination of decision variables, $\alpha_x, \alpha_y \in [0,1]$, $H \geq 0$, and $N_x, N_y \in \{1,2,\dots\}$, that minimizes the total system costs for the proposed transit system.

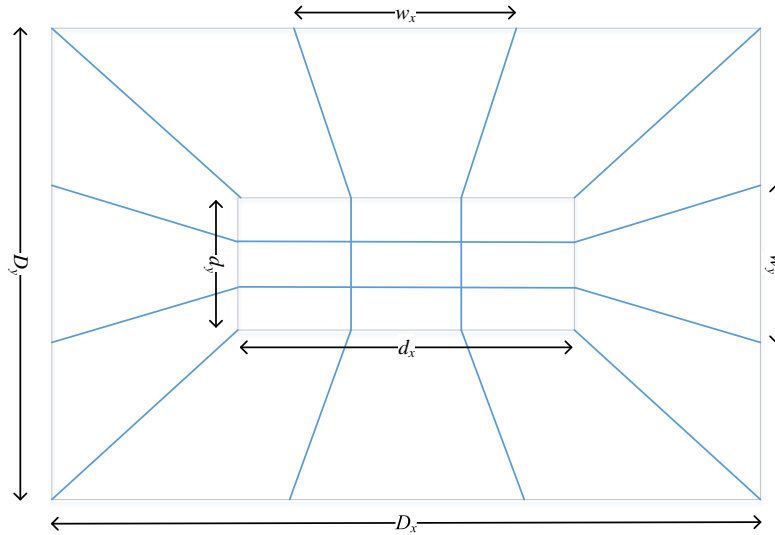


Figure 7.2 General scheme of the structured flexible transit system.

7.2.1 Agency costs

The agency costs include the expected total vehicle distance traveled per hour of operation, Q , and the expected total fleet size in operation, M . The total vehicle distance traveled per hour of operation is as follows.

$$Q = Q_c + Q_p, \quad (1)$$

$$Q_c = \frac{1}{H} \left[\alpha_x N_y D_x + \alpha_y N_x D_y + \frac{2\lambda H \alpha_x D_x (\alpha_y D_y)^2}{3N_y} + \frac{2\lambda H \alpha_y D_y (\alpha_x D_x)^2}{3N_x} \right], \quad (2)$$

$$Q_p = \frac{1}{H} \left[(1 - \alpha_x) N_y D_x + (1 - \alpha_y) N_x D_y + 2\lambda H D_x D_y (1 + \alpha_x)(1 - \alpha_y) l_{px} + 2\lambda H D_x D_y (1 + \alpha_y)(1 - \alpha_x) l_{py} \right], \quad (3)$$

where

$$l_{px} = \begin{cases} \frac{(1 + \alpha_y) D_y}{6N_y} + \frac{2N_y^3}{(1 + \alpha_y)^3 D_y^3 \lambda^2 H^2} - \frac{4N_y^5}{3D_y^5 (1 + \alpha_y)^5 \lambda^3 H^3}, & \text{if } D_y^2 (1 + \alpha_y)^2 \lambda H \geq 2N_y^2 \\ \frac{N_y}{D_y (1 + \alpha_y) \lambda H}, & \text{otherwise} \end{cases}, \quad (4)$$

$$l_{py} = \begin{cases} \frac{(1 + \alpha_x) D_x}{6N_x} + \frac{2N_x^3}{(1 + \alpha_x)^3 D_x^3 \lambda^2 H^2} - \frac{4N_x^5}{3D_x^5 (1 + \alpha_x)^5 \lambda^3 H^3}, & \text{if } D_x^2 (1 + \alpha_x)^2 \lambda H \geq 2N_x^2 \\ \frac{N_x}{D_x (1 + \alpha_x) \lambda H}, & \text{otherwise} \end{cases}. \quad (5)$$

The bus fleet size, M , is simply given by

$$M = Q / v_c, \quad (6)$$

where the bus average travel speed v_c is given by

$$\frac{1}{v_c} = \frac{1}{v} + \frac{2\tau \lambda D_x D_y H (1 + \alpha_x \alpha_y) / (N_x + N_y)}{QH / (N_x + N_y)}. \quad (7)$$

7.2.2. User costs

User costs are related to the total time for passengers to travel from their origin to their destination. Equations (8) – (11) show the expected number of transfers, e_T , the expected waiting time, w , the expected travel time, T , and the expected in-vehicle travel distance, E , respectively.

$$e_T = \alpha_x^2 \alpha_y^2 \left(1 - \frac{1}{N_x} - \frac{1}{N_y} + \frac{1}{N_x N_y} \right) + \alpha_x \alpha_y (1 - \alpha_x \alpha_y) \left(2 - \frac{1}{N_x} - \frac{1}{N_y} \right) + (1 - \alpha_x \alpha_y)^2 \left(\frac{3}{2} - \frac{1}{2N_x} - \frac{1}{2N_y} \right), \quad (8)$$

$$W = \frac{H}{2} (e_T + 1), \quad (9)$$

$$T = \frac{E}{v_c}, \quad (10)$$

$$E = \rho_p E(R_p) + \rho_c E(R_c), \quad (11)$$

where

$$E(R_p) \approx \frac{8}{D_x D_y} \int_{\frac{\alpha_y D_y}{2}}^{\frac{D_y}{2}} \int_0^{\frac{(1-\alpha_x)D_x}{(1-\alpha_y)D_y} \left(y - \frac{\alpha_y D_y}{2} \right) + \frac{\alpha_x D_x}{2}} \left(y - \frac{\alpha_y D_y}{2} \right) dx dy + \frac{8}{D_x D_y} \int_{\frac{\alpha_x D_x}{2}}^{\frac{D_x}{2}} \int_0^{\frac{(1-\alpha_y)D_y}{(1-\alpha_x)D_x} \left(x - \frac{\alpha_x D_x}{2} \right) + \frac{\alpha_y D_y}{2}} \left(x - \frac{\alpha_x D_x}{2} \right) dy dx,$$

$$E(R_c) = \left[\alpha_x^2 \alpha_y^2 \right] \left[\frac{\alpha_x D_x}{3} + \frac{\alpha_y D_y}{3} \right] + \left[\alpha_x \alpha_y (1 + \alpha_x) (1 - \alpha_y) \right] \left[\frac{\alpha_x D_x}{3} + \frac{\alpha_y D_y}{2} \right] + \left[\alpha_x \alpha_y (1 + \alpha_y) (1 - \alpha_x) \right] \left[\frac{\alpha_x D_x}{2} + \frac{\alpha_y D_y}{3} \right] + \left[(1 + \alpha_x)^2 (1 - \alpha_y)^2 / 4 \right] \left[\frac{\alpha_x D_x}{3} + \frac{\alpha_y D_y}{4} \right] + \left[(1 + \alpha_y)^2 (1 - \alpha_x)^2 / 4 \right] \left[\frac{\alpha_x D_x}{4} + \frac{\alpha_y D_y}{3} \right] + \left[(1 - \alpha_x^2) (1 - \alpha_y^2) / 4 \right] \left[\frac{\alpha_x D_x}{2} + \frac{\alpha_y D_y}{2} \right]$$

$$\rho_c = \frac{Q_c H}{4(\alpha_x D_x + \alpha_y D_y)(N_x + N_y)} \text{ and } \rho_p = \frac{Q_p H}{4((1 - \alpha_x)D_x + (1 - \alpha_y)D_y)(N_x + N_y)}.$$

7.2.3 Design

Similar to the flexible route transit system design for a square service area, the total system cost is minimized. The optimization problem is to find the best decision variables α_x (horizontal central rectangle size ratio), α_y (vertical central rectangle size ratio), N_x (number of tubes in horizontal direction), N_y (number of tubes in horizontal direction), and H (bus headway).

The agency cost components are converted to units of user cost (time) by applying the coefficients $\pi_Q = \frac{\$Q}{\lambda D_x D_y \mu}$ and $\pi_M = \frac{\$M}{\lambda D_x D_y \mu}$ (Daganzo, 2010) where $\$Q$ is the agency operation cost per vehicle-distance, $\$M$ is the agency cost per vehicle hour, and μ is the average monetary value of one passenger-hour. Similar to Chapter 3, we let δ measure transfer discomfort, and then $\frac{\delta}{v_w}$ converts the expected transfer number e_T into passenger riding time. The optimization problem becomes the following:

$$\begin{aligned} \text{Min } z &= \pi_Q Q + \pi_M M + W + T + \frac{\delta}{v_w} e_T \\ \text{s.t., } \alpha_x &\in [\frac{1}{N_x}, 1], \alpha_y \in [\frac{1}{N_y}, 1], H \geq 0, N_x \in \{1, 2, \dots, \lfloor D_x / s \rfloor\}, N_y \in \{1, 2, \dots, \lfloor D_y / s \rfloor\}. \end{aligned} \quad (12)$$

In order to find the optimal value of the objective function, z , we may allow N_x and N_y to take a continuous value and apply numerical nonlinear optimization methods. Numerical approximation is used to estimate the gradient of the objective function.

7.3 Input data and proposed design

We obtain the demand data information directly from CUMTD agency. Demand varies by month, day of week, and time of night. For example it varies from 5,096 to 8,182 passengers per month based on the 2013-2014 school year data. Therefore the daily demand varies between 170 and 292 per person. Note that there is higher demand on Friday and Saturday. On average, the busses operate between 7 and 9 hours per night. We can assume 33 passengers per hour per city, which is equal to 4.8 passengers per hour per km². There is no data related to the pick up or drop off locations of safe-ride users since the system is completely manually operated; therefore the CUMTD evening route ridership data is used (Figure 7.3). Figure 7.3 shows the entire collected evening demand points and the rectangle shows the service area that includes majority of the demand points.

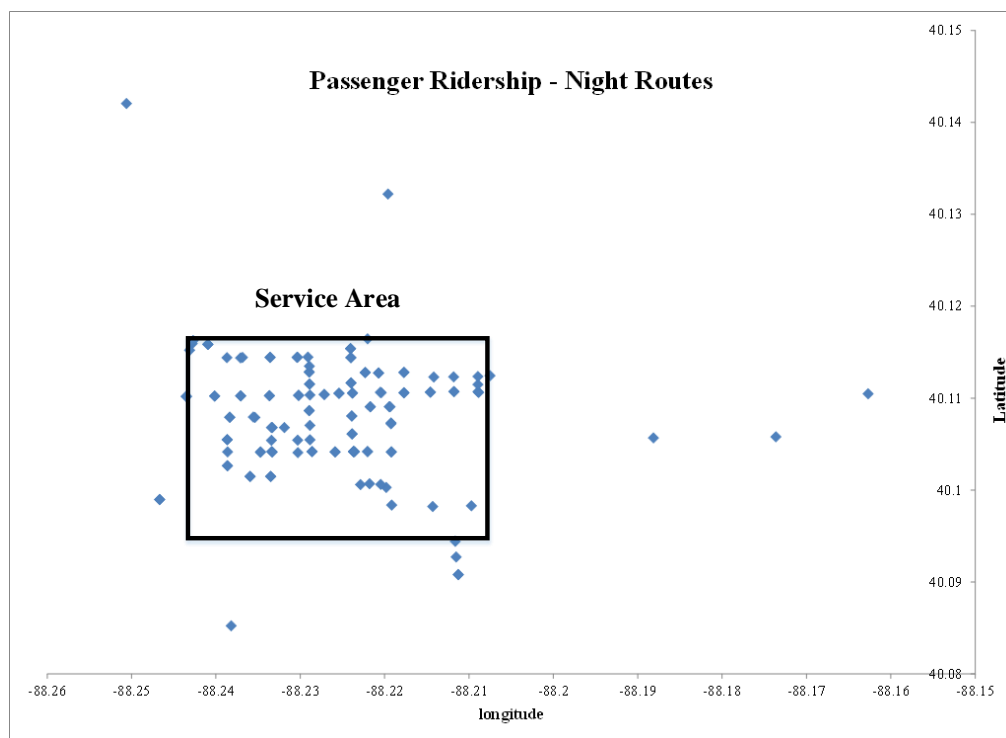


Figure 7.3 Latitude and longitude of the night (late evening) routes ridership.

The service area, chosen based on the demand distribution (Figure 7.3) and the CUMTD service area (Figure 7.1), consists of the UIUC campus, graduate and undergraduate on-campus housing, and a majority of the off-campus housing. The service area is slightly smaller than the CUMTD area since an area without demand was eliminated. As shown in Figure 7.3 the north, the south, east and west borders of the area are respectively University Avenue, Florida Avenue, Vine Street, and Neil Street. The length of the area is 3.3 km (2.5 mile) and width of 2.1 km (1.25 mile). Therefore the total service area is 6.9 km^2 (2.56 mile²).

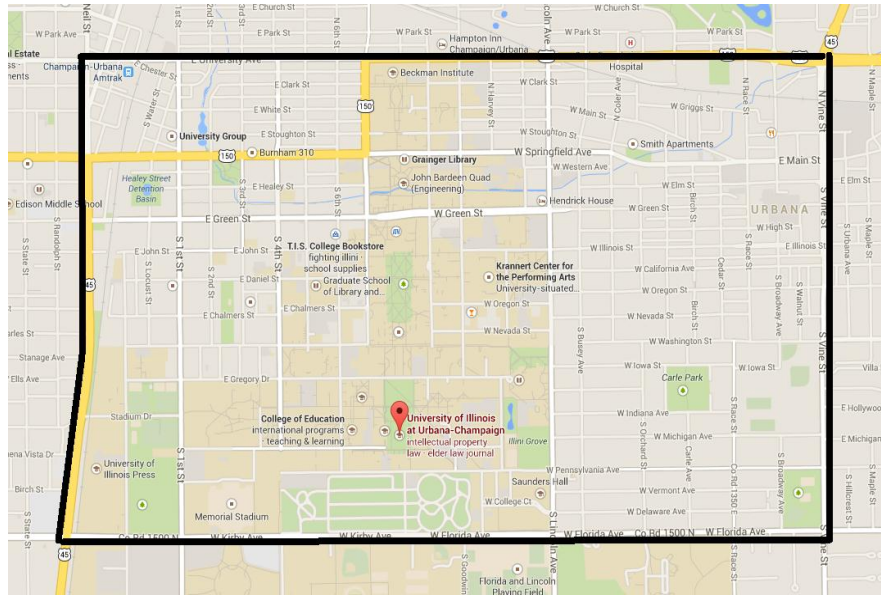


Figure 7.4 Selected service area.

Values for the other parameters are as follows: stop time, $\tau = 13$ sec; city block spacing, $s = 0.15$ km; passenger average time value, $\mu = 20$ \$/hr; bus speed, $v = 25$ km/hr; passenger walking speed, $v_w = 2$ km/hr; transfer discomfort penalty, $\delta = 0.03$; the agency operation cost per vehicle distance, $\$_Q = 2$ \$/veh-km, and the agency operation cost per vehicle time, $\$_M = 40$ \$/veh-hr. Upon

minimization of the total cost, including user and agency cost, the best configuration of the system was determined (Table 7.1).

Table 7.1 Network design and cost components for the flexible-route transit system.

λ	α_x	α_y	N_x	N_y	H (hr)	v_c	$\$Q$	$\$M$	W	T	$\delta \frac{e_T}{v_w}$	z
1	0.91	0.91	2	2	0.10	24.9	2.70	2.16	0.12	0.08	0.02	5.09

As shown in Table 7.1, the optimum network design includes 4 bus tubes (2 in N-S and 2 in E-W directions) with approximately 7 minute headways. The inner part is approximately 80% of the total area (Figure 7.5).

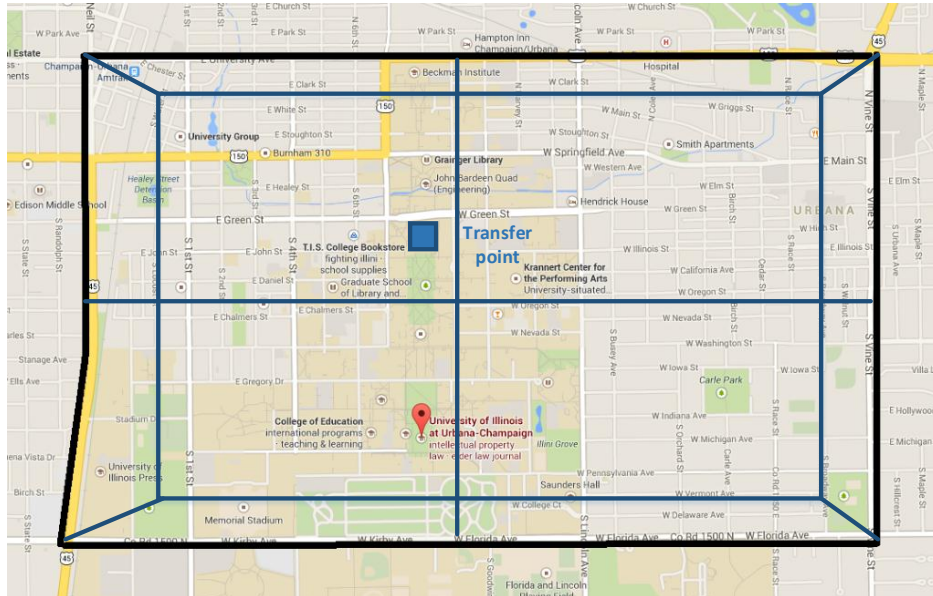
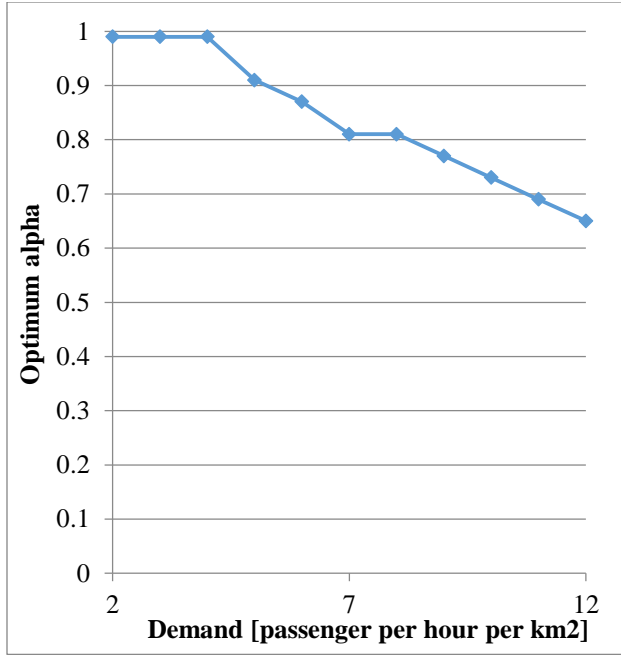


Figure 7.5 Proposed transit system.

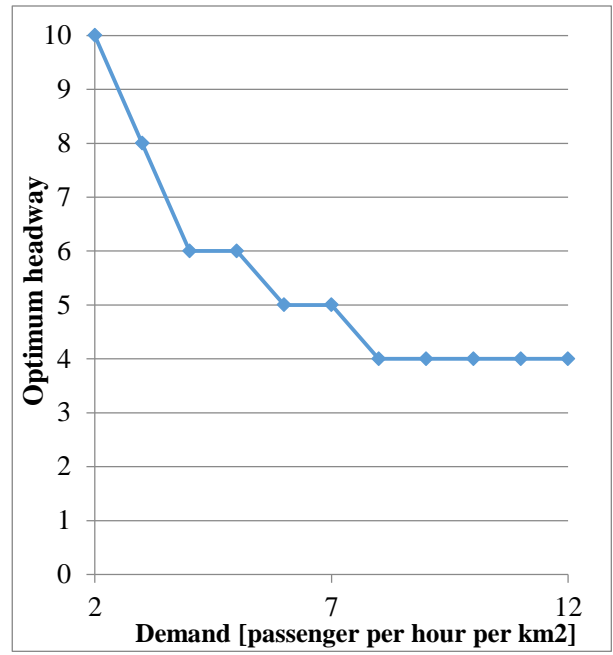
One of the key issues for safe-rides is the ability for passengers to transfer between buses. The proposed methodology allows for designs of one to four transfer points. The most

appropriate location for a single transfer point is near the center of the service area (e.g., Illini union or main library). A single transfer point allows passengers of all four buses to transfer simultaneously, thereby reducing confusion among passengers and drivers. However, the single transfer point causes busses to incur more, but relatively negligible, travel distance. There are also possibilities for two, three or four transfer points.

A sensitivity analyses is conducted to illustrate the effect of demand levels on optimum decision variables. As demand increases, the size of the central part shrinks. Having higher number of passengers gives higher weight to passengers cost vs. agency cost in objective function, therefore the optimum structure of the system has a smaller inner part in order to reduce the user time (Appendix B). Similarly, the optimum headway decreases to reduce passenger waiting time. Figure 7.6 illustrates the variation of optimum alpha and headway vs. demand. For all the demand levels analyzed, the number of buses in each direction remains the same mainly because demand levels in the sensitivity analysis are not high enough to require more bus tubes.



7.6 (a)



7.6 (b)

Figure 7.6 Sensitivity analysis of demand vs optimum (a) alpha, (b) headway.

7.4 Simulation Study

In order to verify and validate the analytical results, we conduct a simulation study. Simulation has some advantages and disadvantages compared to analytical solution. The main benefits of simulation are better understanding of the system by mimicking its behavior and observing the interactions and relations between different system components.

In this section, we simulate the proposed transit system for Champaign-Urbana during late evening and night time periods. We simulate the proposed optimum network using the analytical techniques in Section 7.2 with a minor simplification. The network layout, service area of each bus, and buses headway are known. The origins/destinations are assumed uniform in analytical

analysis. In addition to simulation using uniform demand, we simulate the system using the observed demand. We measure, report and analyze the user and agency cost components of the system. Finally, we compare the performance of the flexible transit system under these two demand patterns.

7.4.1. Assumptions

For the purposes of simulation, we make several assumptions similar to those made in Chapter 3. Trip generation assumptions include that the origin and destination distributions either follow uniform or observed demand distributions and that transit trips occur according to homogeneous Poisson process over time. Additionally, passengers are assumed to be rational hence they use the shortest path, use the minimum number of transfers, and transfer at first opportunity if transfer is needed. Transit system assumptions are as follows. The transit system includes N-S and E-W hemispheres, each containing N equal transit tubes; therefore, there are N transfer points in each tube. All buses have the same headway and sweep longitudinally with lateral movements in their service regions. Since we assume infinitesimal local street spacings and are considering flexible transit, passengers get exact pick-up and drop-off service. We assume one stop of transit per one passenger, as well as stops at transfer locations.

Along with the assumptions described above, the low demand condition is an essential presumption for simulation. The average hourly demand for the entire UIUC campus is 33 passengers that is considered as very low demand. Based on the numerical results in section 3.4.1 flexible transit system can provide lowest cost for Champaign-Urbana over late evening and night. The system cost components for both agency and user are assumed similar to the analytical part, Section 7.2.

7.4.2. Transit Demand Distribution

Maintaining the same service area regardless of demand distribution, we use Monte Carlo simulation method to generate the demand for transit based on two demand patterns: uniform and observed demand distributions.

For uniform demand, we generate random origins independently of random destinations. We generate a random point inside the service area by first generating normalized coordinates, two independent uniform numbers between 0 and 1, then multiplying by region's length and width, respectively. The number of generated origins and destinations are based on the average night demand. These origins and destinations are randomly paired.

Another simulation case is based on the observed demand. In this case we use the CUMTD evening ridership data (Figure 7.3). The passengers board and alight at existing bus stations. We consider the locations of these bus stations as origins and destinations. A map service (e.g., google map) is utilized to find the longitudes and latitudes of all stops. Then we locate these global coordinates within the service region. We adjust the total number of generated trips to 33 passengers per hour which is the average night demand. Then these origins and destinations are also randomly paired.

Figure 7.7 (a) and 7.7 (b) respectively show the location of origins/destinations of uniform and observed distributions. Origins/destinations are more scattered in Figure 7.7 (a) because they could be located at any position inside the service area; however in Figure 7.7 (b) that follows the observed CUMTD ridership, they are located at the existing bus stops.

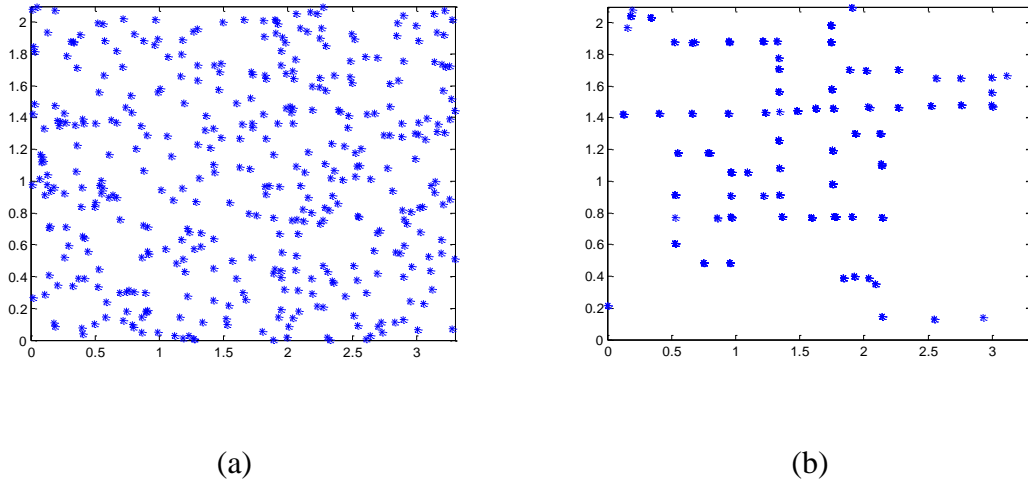


Figure 7.7 Origins/Destinations locations (a) uniform demand, (b) observed demand.

After generating origins/destinations locations pairs, the next step is to start trip assignment. For both scenarios we generate trip start times based on the homogeneous Poisson process. The time interval between two trip start times, Δt , can be obtained using equation (13).

$$\Delta t = \frac{-\ln(u)}{33}, \quad (13)$$

where u is the a uniform random number between 0 and 1, and 33 is the hourly demand of the service area. These generated trip start times are randomly assigned to generated origins/destinations locations pairs.

We set $\alpha_1 = \alpha_2 = 1$ that gives us grid transit network. These values are very close to the optimum α_1 and α_2 that both are equal to 0.91. This assumption simplifies the given network structure and avoids complexity without getting far from the optimum design. We set N , the number of tubes or the number of transfer points in each directions, to be 2 and H , headway, 0.15

dispatching any bus for that period. The program keeps track of the start time of the trip, pick up time, drop off at transfer point, pick up from transfer point and drop off at the destination in order to obtain the different user cost components. It also tracks every bus travel distance and time to get the agency cost components.

7.4.3. Simulation results

The simulation was coded using MATLAB. The code runs on a computer with 2.9 GHz CPU and 8 GB RAM. Running time for each case is about a 5 seconds. The model is run 10 times based on different random seeds. For each run the duration of bus service is 12 hours. In order to capture the performances of the system in the steady state, we start collecting data one hour after the start of dispatching the first bus and stop one hour before the last bus service. Thus the effective duration of simulation time for each run is 10 hours. During each run the average number of served passengers is 330.

During the simulation, we capture the time of occurrence of different events. For example, Figure 3 illustrates a sample micro simulation result. In this figure the columns 1 to 12 indicate the passenger number, trip start time, origin and destination horizontal and vertical coordinates, assigned first and second bus if any, pick up and drop off time by the first bus, and pick up and drop off time by the second bus if a transfer is needed, respectively.

	1	2	3	4	5	6	7	8	9	10	11	12	13
87	87	2.8617	0.2022	1.1246	1.6978	0.8072	5	4	2.8797	2.9923	3.0246	3.0900	
88	88	2.9194	0.0390	1.4252	2.2613	0.1247	2	7	3.0620	3.1331	3.1866	3.2637	
89	89	2.9291	2.8832	0.5065	0.0463	0.7112	8	0	3.0538	3.2133	0	0	
90	90	2.9347	2.9842	0.1419	0.6710	0.1243	8	0	3.0316	3.1612	0	0	
91	91	2.9836	2.7812	1.1056	2.5437	1.2482	6	0	3.0431	3.0620	0	0	
92	92	3.0208	0.0507	1.9392	0.0827	2.0188	1	0	3.1121	3.1202	0	0	
93	93	3.0221	0.1450	1.2767	0.0904	2.0325	6	0	3.1627	3.1987	0	0	
94	94	3.0554	1.3348	1.3192	2.5173	1.5901	5	0	3.0672	3.1290	0	0	
95	95	3.1069	1.6125	1.6725	2.0167	0.3899	2	7	3.2058	3.3330	3.3366	3.4081	
96	96	3.1235	1.5633	1.7428	0.2930	0.5759	2	0	3.1974	3.3061	0	0	
97	97	3.1567	1.6095	1.2960	3.1901	1.7427	5	0	3.2292	3.3139	0	0	
98	98	3.1995	0.5280	1.8480	2.9884	0.1020	5	4	3.3356	3.4385	3.4746	3.5649	
99	99	3.2053	0.8605	1.8169	0.5753	1.0319	2	0	3.3164	3.3628	0	0	
100	100	3.2352	1.7729	0.4237	1.4107	0.6021	8	0	3.3687	3.3940	0	0	
101	101	3.2556	1.3552	1.9338	2.7923	0.5369	5	4	3.3758	3.4422	3.4782	3.5360	
102	102	3.3140	0.9911	0.2493	2.6982	1.4228	7	3	3.3579	3.4355	3.4746	3.5231	

Figure 7.9. Sample micro simulation result.

Using all detailed information of the events, we can extract different cost components. For example by subtracting the passengers' drop off from their pick up time at the transfer points we obtain the transfer waiting time. Table 1 shows the aggregated results of the uniform and observed demand distribution as well as the analytical results for different user cost components.

Table 7.2. Cost components.

	Simulation (uniform demand)	Simulation (observed demand)	Analytical (uniform demand)
Initial waiting time [hour]	0.079	0.079	0.075
Transfer waiting time [hour]	0.066	0.061	0.075
Total waiting time [hour]	0.094	0.095	0.120
In vehicle travel time [hour]	0.085	0.068	0.080
User cost [hour]	0.190	0.163	0.200
Expected # transfers	0.250	0.260	0.250 ¹⁴
Vehicle distance travel [km/hour]	85.06	81.08	173.7 ¹⁵

¹⁴ Expected number of transfer for ideal grid network

¹⁵ Total vehicle distance travel per operation hour is calculated for grid network

In the analytical solution, we assume the mean waiting time is simply half of the headway. However, in the observed demand the waiting time is random and more likely greater than the analytical assumptions because of randomness. In cases where there are more passengers than average, it takes more time for the bus to serve them which causes extra delay due to passenger accumulation. This makes the waiting time especially for downstream passengers more than $H/2$. Since the demand is relatively low, no severe delays happen and the average waiting time is a slightly higher or equal to $H/2$. Note that the analytical results are for the system with $\alpha=0.91$ and therefore the number of passengers who transfer are higher. To better investigate the waiting time, Figure 7.10 (a) and 7.10 (b) illustrate the initial waiting time for uniform and observed demand. As it shown in the figures, the waiting time is randomly distributed between 0.003 and 0.21 for both cases. The average waiting time for both cases is 0.079 and the variance is 0.0021 and 0.0022 for uniform and observed demand, respectively. In both cases the average and variance of initial waiting time don't show major difference.

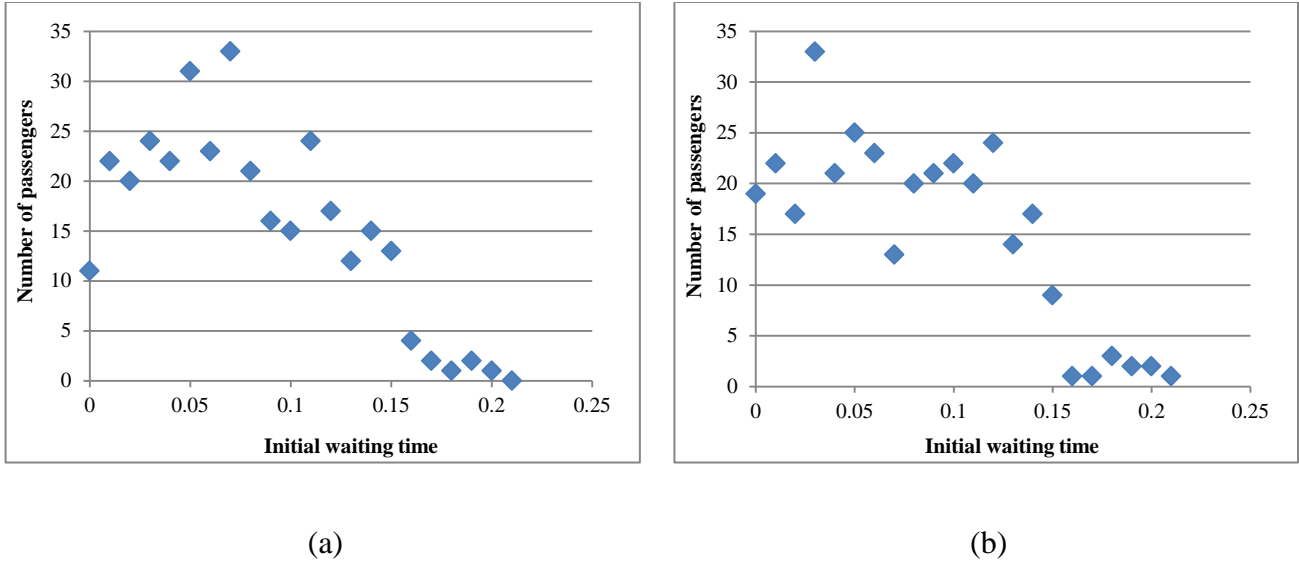


Figure 7.10. Passenger's initial waiting time for (a) uniform and (b) observed demand.

The average travel time in the simulation study using uniform demand is higher than the analytical results since the studied area is grid and the analytical study area is hybrid in which the travel time is lower (Appendix B). However the observed demand case results in less travel time because the corresponding origins and destinations are not scattered throughout the whole service area (Figure 7.7.a).

In addition we analyze the agency cost components. The total vehicle distance travel per hour of operation is 85.06 and 81.08 km for uniform and observed demand distributions respectively. These numbers are both less than the analytical results because in the simulation we never dispatch buses if there is no need for pick up or drop off passengers. This assumption is different with the analytical assumption. The average demand level in this case study is very low and occasionally there is insufficient demand to warrant the dispatch of a bus. The uniform demand has higher travel distance because both origins and destinations are more sparsely distributed.

7.5 Conclusion

This chapter conducts a real-world case study for a flexible-route transit network. The general concepts of the system includes home-to-home pick up and drop off similar to the system presented in chapter 3. Instead of square, the service area was generalized to a rectangle and closed-form formulas for user and agency cost were re-derived. The optimization problem determines the optimum size of grid part, tube sizes in horizontal and vertical directions and bus headways.

The proposed design for Champaign-Urbana's safe-ride system can replace the current manual procedure. This proposed system is easy to use by passengers and drivers. Currently there are three transfer points; however, the proposed design reduces the transfer points to a convenient one near the center of campus. Although the current system can effectively estimate the expected waiting time, the proposed system eliminates the potential for a long waiting time. Furthermore, unlike the current procedure, the proposed design estimates user cost components (e.g., waiting or in vehicle travel time) and allows for better resource allocation.

We conduct simulation to understand how the proposed transit system might work in reality. Simulation also helps verify and validate the analytical results. We conduct the study on the simplified proposed optimum network using both uniform and observed demand distributions. The micro simulation results show detailed information for every passenger and the aggregated result shows the system cost components for each scenario. The obtained user and agency cost components not only validate the analytical results but also provided intuition. Simulation results

also confirm that the difference between uniform and observed demand is not considerable in different cost components.

Chapter 8

Real-world case study: Weihai

8.1. Introduction

Chapters 4, 5 and 6 proposed transit network design for the system with heterogeneous demand distribution and grid or polar shape. Those design techniques were applied to examples with hypothetical demand distribution; however it can also be applied to real-world cases. To conduct a real-world case study, we obtained transit demand data from the current transit network of Weihai city in northeastern part Shandong Province of China, which has an urban population of 591,000¹⁶. Figure 8.1 *Weihai, Shandong, China* shows the city of Weihai.

¹⁶ <http://en.wikipedia.org/wiki/Weihai>

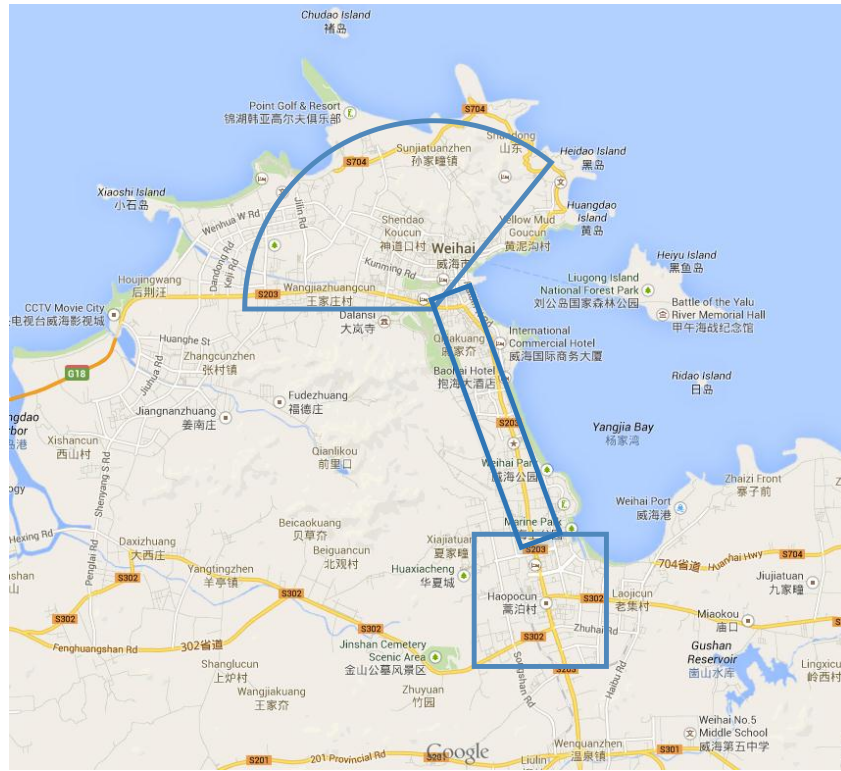


Figure 8.1 Weihai, Shandong, China.

This coastal city can be divided into two regions and a connecting corridor as it is shown in Figure 8.1. We analyze the demand data and propose suitable transit network for each part of the city. Based on the shape of the network and the structure of this city, we postulate that a polar network design for the northern part and grid network for the southern part will be the most suitable. In addition, there are two major roadways connecting these two parts that we consider in our design. In this chapter we design a transit system for Weihai city using the design techniques proposed in Chapters 5 and 6.

The organization of this chapter is as follows. After this brief introduction, in the next Section we describe how we obtain the demand distribution data. In Section 8.3 we analyze the system and propose optimum design for all different parts of the city. The last section concludes and summarizes this chapter.

8.2 Demand analysis

The first step in transit network design is to determine the transit demand distribution. We obtain the demand distribution of the city based on data from the existing transit system. Figure 8.2 *Current transit network of Weihai city* illustrates the city transit system in year 2009. We consider annual growth factor of 1% and design transit system based on the projected demand in the year 2020. The current demand data include boarding and alighting passengers at all stations. Therefore for each bus we can calculate the onboard passengers in both directions. Therefore for each neighborhood we can obtain the trip production, trip attraction, approximate the flux in each direction.

Figure 8.3 shows the latitude and longitude of all the 2,736 pints. Note that in the original data there is a lot of missing information (e.g., headway, boarding and alighting incompatibility) so we make proper assumptions about them.

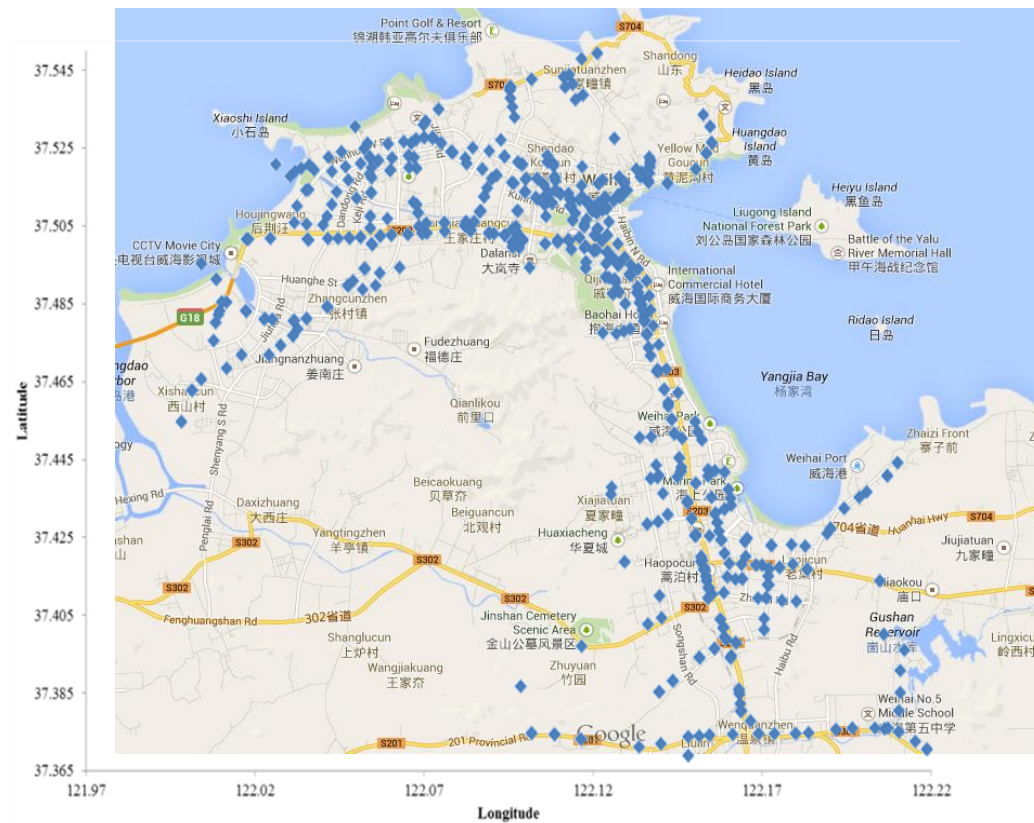


Figure 8.3 Collected bus stops data.

8.3 Proposed transit system

In order to design transit network for Weihai, we divided the city to three parts with different structure patterns. The upper part is mostly residential, which forms a pie with 9 km radius and 120 degree angle. The south part is Weihai downtown (CBD), which forms a grid structure with approximately 4 km length and width. The third corridor is the connection between the

residential and commercial part of city, which forms a rectangle with approximately 8 km length and 500 m width. In this rectangle there are one major and one local highway in N-S direction. Note that even though we are designing each section separately, we are still considering the global demand distribution for the entire city. The design procedure is an iterative algorithm which chooses a consistent optimum headway that minimizes the total cost of the three parts of the city.

8.3.1 Polar part

For the polar part we optimize the radial separation degree, ring spacing, and bus headway. The polar design part in chapter 6 was proposed for a complete circular shape city. This city is not a complete circle but a pie with 120 degree angle with total area of 27π . Table 8.1 shows the latitude and longitude of the center, western and northern corner of the pie. The demand is considered uniform over the space with an average of 78.42 passengers per km^2 per hour.

Table 8.1 Latitude and longitude of the polar part.

	center	West corner	North corner
Latitude	37.50	37.501	37.554
Longitude	122.124	122.019	122.142

Table 8.2 shows the parameter settings for the design. All the assumptions and notations are the same as chapter 6.

Table 8.2 Input parameter settings of the model.

Parameter	v (km/hr)	v_w (km/hr)	τ (s)	c_s (\$/bus- stop)	c_V (\$/bus- km)	c_M (\$/bus- hr)	R (km)	μ (\$/hour)	$disc$ (min)
Value	25	2	30	0.3	2	40	9	15	1

The optimum network design considering consistent headway for the entire city is applied to the network. Table 8.3 shows the optimum design and associated user and agency costs.

Table 8.3 Optimum decision variables and cost components for polar part.

	Optimum design
H (min)	9
R (km)	0.3-1.7
Θ (radian)	0.2
Agency costs (time/hour)	846
User costs (time/hour)	6,030
Total costs (time/hour)	6,876

Current design follows grid pattern in eastern part with horizontal and vertical spacings between 0.5 km and 1.0 km. In the western and north eastern part, the design follows the radial and ring

with 0.7 km and 0.2 km, respectively. Our design is based on uniform demand which leads to uniform radial spacings of 0.2 radian. The ring spacing varies between 0.3 km and 1.7 km. Both horizontal and vertical spacings need adjustment based on the actual demand and street pattern. Also in some central part with grid street pattern, we can follow the street pattern by twisting radial-ring spacings.

8.3.2 Grid part

The downtown is located at the southern part of the city. The street pattern in this area is grid and we design grid transit network for this part. We select a square with approximately 4 km each side that represents most of the current bus stops. All latitude and longitude of the square are shown in Table 8.4. The average total demand in the entire grid is 4,884 passengers per hour. For this area, we consider the optimization of the horizontal and vertical bus spacing, as well as headway.

Table 8.4 Latitude and longitude of the grid part.

	North- west	North- East	South- West	South- East
Latitude	37.436	37.436	37.400	37.400
longitude	122.132	122.182	122.132	122.182

Table 8.5 shows the parameter settings. The assumptions, notations and parameters are the same as Chapter 5.

Table 8.5 Input parameters of the model.

Parameter	v (km/hr)	v_w (km/hr)	τ (s)	c_s (\$/bus- stop)	c_V (\$/bus- km)	c_M (\$/bus- hr)	L (km)	W (km)	μ (\$/hour)	θ (min)
Value	25	2	30	0.3	2	40	10	10	15	1

The optimum network design considering consistent headway for the entire city is applied to the network. Table 8.6 shows the optimum design and associated user and agency costs.

Table 8.6 Optimum decision variables and cost components for grid part.

H (min)	9'
Vertical Spacings	0.7-1.9
Horizontal Spacings	0.4-1.9
Agency costs (time/hour)	246
User costs (time/hour)	4006
Total costs (time/hour)	4252

As shown in the table the global optimum headway is 9 minutes. The vertical spacing varies between 0.7 km and 1.9 km and the horizontal spacing varies between 0.4 km and 1.9 km. Current vertical and horizontal spacings are between 0.75 km and 1.4 km, and 0.8 km and 1.3

km, respectively. Knowing the optimum spacings, adjustment based on the current city geometry is needed.

8.3.3 Connection corridor

The connection area locates in the middle part of the city (west side of the sea and east side of the mountain) and connects residential area to CBD. This corridor is approximately a rectangle with about 8 km length and 500 m width. Table 8.7 shows latitude of longitude the mid-north and south points of the corridor.

Table 8.7 Latitude and longitude of the connection corridor.

	Mid-North	Mid-South
Latitude	37.50	37.426
longitude	122.124	122.157

In this part we are trying to find the optimum headway, H , and stop spacing, s , for two transit lines. Let's assume the transit line is L and set a , b , and F as the average alighting, boarding and on-board passengers in the bus, respectively. Also let's set a' and b' as total alighting and boarding in the corridor. We can formulate the associated total users and agency costs as follows:

$$Q = \frac{2N}{H}[L], \quad (1)$$

$$M = Q / v_c, \quad (2)$$

$$\frac{1}{v_c} = \frac{1}{v} + \frac{\tau}{s}. \quad (3)$$

$$W = a' \frac{H}{2}, \quad (4)$$

$$A = (a' + b') \left(\frac{s}{4} + k \right), \quad (5)$$

$$E = \frac{NL}{3}, \quad (6)$$

$$T = a' \frac{E}{v} + F \frac{\tau NL}{s} \quad (7)$$

The optimization problem is to find the

$$\text{Min } z = \frac{\$Q}{\mu} Q + \frac{\$M}{\mu} M + W + T + A \quad (8)$$

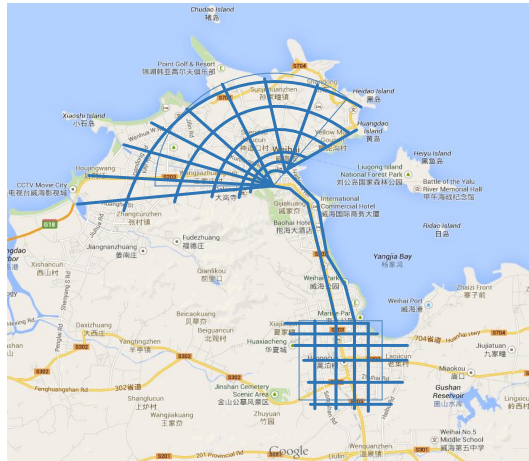
s.t., $s, H \geq 0$.

This problem can be formulated as the general EOQ type of problem for stop spacings. We solve this problem numerically to obtain the optimum stop spacings along with the optimum headway. The cost components of this part are considered in the optimum headway design. The optimum headway is 9 minutes and the optimum average bus stop spacing is 0.4 km. This spacing could be designed shorter or longer depending on the road geometry.

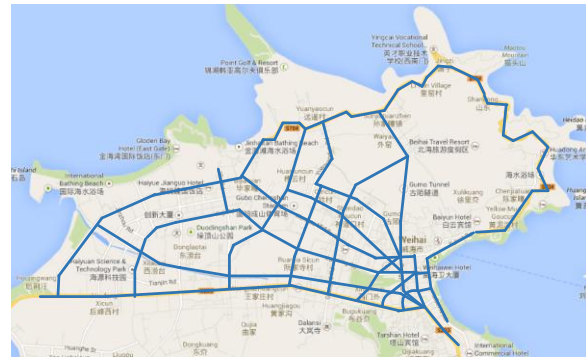
8.3.4 Discussion

This section provides the design for three different parts of the city. We considered the entire city demand distribution even though each part is independently designed. In addition we assume

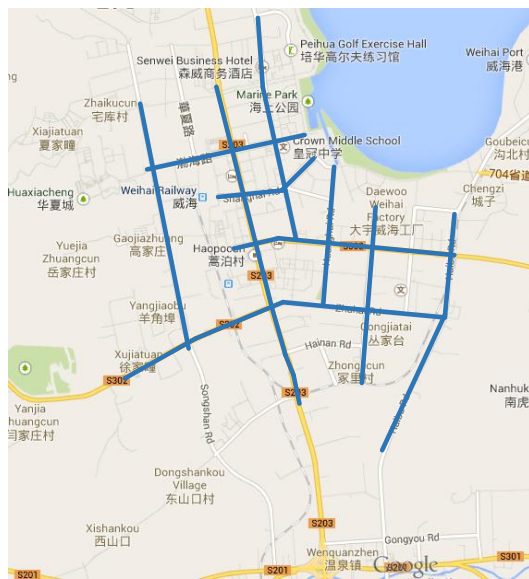
consistent headways for all parts. Figure 8.4 illustrates the general scheme of the final design as well as the proposed detailed transit network layout for different part of the city.



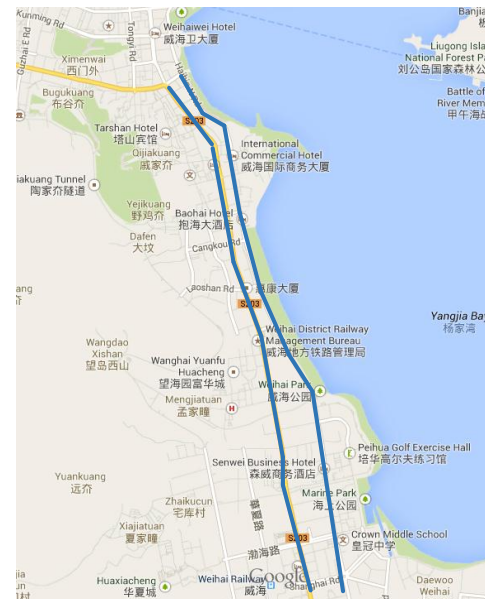
(a)



(b)



(c)



(d)

Figure 8.4 (a) Design overview for the entire city; detailed transit network layout of (b) polar, (c) grid, and (d) the connection corridor areas.

We obtained the demand density of this city by using its existing transit system. This is not the best way to obtain the demand. More advanced demand collection method is needed for more accurate design.

The optimization problem is the tradeoff between agency and user cost. The higher the agency cost components (e.g., driver cost), the higher the headway and the bus spacings. We made proper assumptions about these cost components, however considering exact cost for the specific city may result in slightly different optimal decision variables and transit system.

Since we don't have much information about the existing transit service and there are also lots of missing data, we are not able to make exact comparison.

8.4 Conclusion

This chapter conducts a real-world case study for Weihai in Shandong Province of China. We obtain the demand density of the city using the current transit system of this city. We divide the city into three parts based on the street layout and the geometrical shape of service area: polar area, grid area and connection corridor. We use the design techniques in Chapter 5 for the grid area, Chapter 6 for polar area and proposed equations in this chapter for connection corridor. Our design includes finding the best key parameters of transit network such as headway, bus line spacing, and bus stop spacing. Knowing the design key parameters, we can suggest the detailed transit system based on the street layout.

Chapter 9

Conclusion and Future Research Plan

9.1. Conclusion

This Ph.D. dissertation focuses on transit network design for both fixed and flexible route systems in an innovative way. The first chapter introduces the overall structure of the dissertation research and highlights the objectives and contributions. Then we review the existing literature on transit network design that includes traditional approaches and a few innovative transit network design methods. For example, we review the continuum approximation method which is an efficient tool in solving transportation problems.

The first part of this work proposes a flexible-route transit network using a hybrid network structure. The flexible-route buses pick up passengers from their travel origins and drop them off at their destinations, hence eliminating the need for passengers to walk to and from the stations. If needed, passengers can transfer between buses at designated transfer points near the city center. The layout of the transit network includes grids in the central part of the city and a hub-and-spoke network in the peripheral part. We analyze the agency and user cost components and design a transit system that minimizes the total cost. The optimum design determines the layout of hybrid network, width of central grid region and buses headway. This transit system design can be shown to reduce the total cost for low demand levels and provide a safer service to passengers.

In the second part of this PhD dissertation we look at the fixed-route transit network design under heterogeneous spatial demand. In order to handle demand variation over space, we propose to use the CA method for transit network design. In order to design a transit system that

can accommodate the variable demand we present two ideas: transit system with local lines and variable bus line spacings. We use the local lines to provide denser route service in high demand areas. We align local and main buses by using a power-of-two scheme where all local and mainline spacings take values from a geometric scale. The CA method allows us to derive closed-form formulas for all agency and user costs, hence we can design a transit system that minimizes the total cost. The optimum design determines the spacings of main and local bus lines as well as bus headway. The numerical case studies show that under heterogeneous demand, the proposed transit network configuration provides the lower cost compared to regular homogeneous grid transit networks (without local bus lines). In addition, we also propose a simpler grid network in order to address spatial demand variation. Using the CA approach, we analyze both agency and user cost components and obtain closed-form formulas. We design the transit system by selecting the optimum network layout and bus headway that minimizes the total system costs. The numerical examples show that lower costs can be achieved as compared with homogeneous grid transit networks.

In the other part of this dissertation, we propose to extend the idea of heterogeneous demand transit network design as well as the CA method to circular shaped cities with a ring-radial street network. Similarly, we analyze the agency and user costs and design the optimum transit network with circular and radial buses. Hypothetical case studies show how the model can be implemented.

In the last part of this dissertation, we apply the theoretical findings to real-world cases for both flexible- and fixed-route networks. We propose flexible-route transit network for Champaign-Urbana using hybrid structure. Suggested flexible-route transit system can outperform the existing manually dispatched safe-ride system with several advantages. We also

apply the fixed-route transit network design methodology to the city of Weihai in China. The city includes grid and polar network structures as well a corridor that connects them to each other. The optimization problem determines the key decision variables such as bus line spacing, stop spacing and headway.

9.2. Future research

There are many possible extensions to flexible-route and fixed-route transit network design that we can take into consideration during the Ph.D. dissertation research. In both directions more emphasis will be casted on practical applications of the design, particularly on real-world implementation of the proposed design methods.

In the flexible-route transit system design we have proposed a transit network for a square and rectangle city with homogeneous demand. We can generalize the shape of the service region to circle, or some other general shapes. However considering heterogeneous spatial demand distributions or more general system setting, will make it harder to obtain closed-form formulas for the system costs. Therefore, we probably need to consider other analytical approaches. On the other hand, in the design procedure, we discover that the hybrid of hub-and-spoke and grid transit network seem to be a suitable layout for transit networks. Another possible future research direction is to explore additional network structures (e.g, multi-centric hub-and-spoke systems).

Similar to the flexible-route transit system design, we can relax some of the assumptions to generalize the model. In closing, Table 9.1 summarizes the current and possible future work on both flexible-route and fixed-route transit network design.

Table 9.1 Status of current studies in the dissertation and possible future research.

		Current Work	Possible Future Research
Flexible-route Network	Service Region Shape	<ul style="list-style-type: none"> • Square • Rectangle 	<ul style="list-style-type: none"> • General
	Transit Network Layout	<ul style="list-style-type: none"> • Grid • Hub-and-spoke • Hybrid 	<ul style="list-style-type: none"> • Multi hub-and-spoke
Fixed-route Network	Service Region Shape	<ul style="list-style-type: none"> • Rectangle • Circular 	<ul style="list-style-type: none"> • Pie • General
	Transit Network Layout	<ul style="list-style-type: none"> • Grid • Polar 	<ul style="list-style-type: none"> • Multi hub-and-spoke • Hybrid

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Appendix A

Result 0: We list the following useful geometry quantities without proof.

Table A.1 Geometry quantities.

Area of each tube that belongs to the inner square	$\frac{D^2 \alpha^2}{N}$
Area of each tube that belongs to the outer peripheral part	$\frac{D^2(1-\alpha^2)}{2N}$
Expected lateral distance per passenger in the center	$\frac{D\alpha}{3N}$
Longitudinal distance per bus in the central square during one round trip	$2\alpha D$
Longitudinal distance per bus in the peripheral part during one round trip	$2(1-\alpha)D$
Expected number of passengers to pick up and drop off in the central square during a round trip	$\frac{2\lambda H D^2 \alpha^2}{N}$
Expected number of passengers to pick up and drop off in the peripheral part during a round trip	$\frac{2\lambda H D^2 (1-\alpha^2)}{N}$

Result 1: The expected number of transfers is given by this formula: $e_T = \frac{N-1}{N} \left(1 - \frac{\alpha^4}{N} \right) + \frac{(1-\alpha^2)^2}{2}$.

Proof: One trip may include 0, 1 or 2 transfers, depending on the relative locations of the origin and destination. The conditional probability of having zero transfer is $\frac{2}{N} - \frac{1}{N^2}$ if both the origin and destination locations are in the central square (case 1), $\frac{1}{N}$ if one is in the peripheral quadrants and the other is in the central square (case 2), or $\frac{1}{2N}$ if both are in the peripheral quadrants (case 3). Therefore the unconditional probability for a trip to have zero transfer is

$$\Pr[e_T = 0] = \alpha^4 \left(\frac{2}{N} - \frac{1}{N^2} \right) + 2\alpha^2(1-\alpha^2) \frac{1}{N} + (1-\alpha^2)^2 \frac{1}{2N}.$$

Similarly, it can be shown that the conditional probability of one transfer is $\frac{N-2}{N} + \frac{1}{N^2}$ in case 1, $\frac{N-1}{N}$ in case 2, or $\frac{1}{2}$ in case 3. Therefore,

$$\Pr[e_T = 1] = \alpha^4 \left(\frac{N-2}{N} + \frac{1}{N^2} \right) + 2\alpha^2(1-\alpha^2) \frac{N-1}{N} + (1-\alpha^2)^2 \frac{1}{2}.$$

Two transfers may occur with conditional probability $\frac{N-1}{2N}$ only in case 3; i.e.,

$$\Pr[e_T = 2] = \alpha^4 \times 0 + 2\alpha^2(1-\alpha^2) \times 0 + (1-\alpha^2)^2 \frac{N-1}{2N}.$$

Therefore, the expected number of transfers is $e_T = \Pr[e_T = 1] + 2\Pr[e_T = 2] = \frac{N-1}{N} \left(1 - \frac{\alpha^4}{N} \right) + \frac{(1-\alpha^2)^2}{2}$.

Result 2: The hourly total vehicle distance is given by $Q = Q_c + Q_p$, where

$$Q_c = \frac{2N}{H} \left[\sum_{i=2}^{\infty} \alpha(i-1)DP_c\{i\} + 2\alpha D + \frac{2\lambda HD^3 \alpha^3}{3N^2} \right] \text{ and}$$

$$Q_p = \frac{2N}{H} \left[\sum_{i=2}^{\infty} (1-\alpha)(i-1)DP_p\{i\} + 2(1-\alpha)D + \frac{2\lambda HD^2 (1-\alpha^2)l_p}{N} \right]$$

are the expected distances in the central square and the peripheral part, respectively,

$$P_c\{i\} = \left(\frac{\alpha D s \lambda H}{N} \right)^i e^{-\frac{\alpha D s \lambda H}{N}} / i! ,$$

$$P_p\{i\} = \left(\frac{(1+\alpha) D s \lambda H}{N} \right)^i e^{-\frac{(1+\alpha) D s \lambda H}{N}} / i! , \text{ and}$$

$$l_p = \begin{cases} \frac{(1+\alpha)D}{6N} + \frac{2N^3}{(1+\alpha)^3 D^3 \lambda^2 H^2} - \frac{4N^5}{3D^5 (1+\alpha)^5 \lambda^3 H^3} , & D^2 (1+\alpha)^2 \lambda H \geq 2N^2 \\ \frac{N}{D(1+\alpha)\lambda H} , & \text{otherwise} \end{cases} .$$

Proof: The total vehicle distance per hour is given by the product of the number of bus tubes, the expected travel distance of one bus round trip, and the inverse of the bus headway. The expected travel distance per bus round trip includes three parts: longitudinal distance, lateral distance, and extra detour distance.

In the inner square, the round-trip longitudinal distance is $2\alpha D$ and the lateral distance is $\frac{2\lambda H D^3 \alpha^3}{3N^2}$; in the peripheral quadrants, the round-trip longitudinal distance is $2(1-\alpha)D$ and the lateral distance is $\frac{2\lambda H D^2 (1-\alpha^2) l_p}{N}$, where the expected lateral distance per passenger in the peripheral quadrants, l_p , is computed as follows.

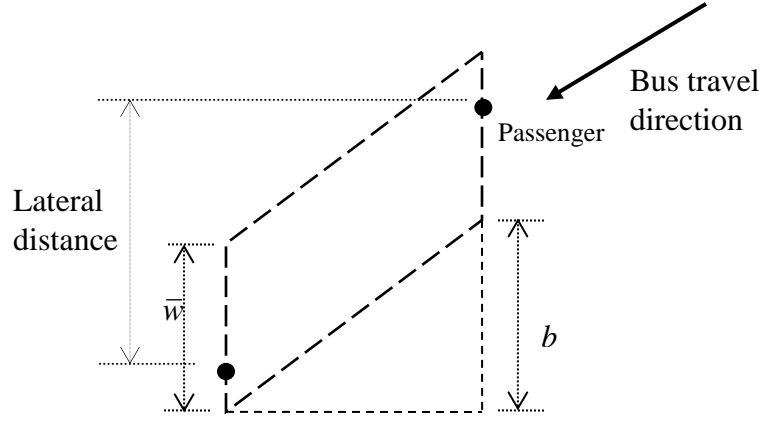


Figure A.1. The expected lateral distance to visit one passenger in a slanted tube.

Note that the tube is slanted in the peripheral quadrants, as illustrated in Figure A1. The expected lateral distance, l_p , between the two dots (i.e., consecutive passengers) can be shown to be as follows:

$$l_p = \begin{cases} \frac{\bar{w}}{3} + \frac{b^2}{\bar{w}} - \frac{b^3}{3\bar{w}^2} & \text{if } b < \bar{w} \\ b & \text{if } b \geq \bar{w} \end{cases}.$$

where \bar{w} is the local tube width and b is the lateral off-set of the tube between the two consecutive passengers. Considering Poisson process for passenger locations and $0^\circ \sim 45^\circ$ slant angles of the tubes, the expected value of b across all local areas of the quadrants is approximately $\frac{1}{w(1+\alpha)\lambda H} = \frac{N}{D(1+\alpha)\lambda H}$, and hence the approximate¹⁷ formula for l_p follows.

Since the street spacing $s > 0$, the bus must travel extra distance if it needs to visit more than one passenger before advancing through one street block in the longitudinal direction; see

¹⁷ The formula for l_p is approximate due to Jensen's Inequality.

Figure A2.¹⁸ The expected extra longitudinal distance to visit each additional passenger is $\frac{s}{2}$.

The number of passengers in the “tube block” area (i.e., the shaded area in Figure A2) follows a Poisson distribution with mean $\bar{w}s2\lambda H$, where $\bar{w} = \alpha D/N$ in the central square (considering double coverage in the center) and $(1+\alpha)D/2N$ in the peripheral quadrants. Therefore the probabilities of having i passengers in the center and peripheral parts are

$$P_c\{i\} = \left(\frac{\alpha D s \lambda H}{N}\right)^i e^{-\frac{\alpha D s \lambda H}{N}} / i! \text{ and } P_p\{i\} = \left(\frac{(1+\alpha) D s \lambda H}{N}\right)^i e^{-\frac{(1+\alpha) D s \lambda H}{N}} / i! \text{ respectively.}$$

If there are $i \geq 2$ passengers in one “tube block”, the bus incurs an extra distance of $(i-1)\frac{s}{2}$ per passenger, and the expected extra distance per round trip is therefore $\alpha D(i-1)P_c\{i\}$ in the center part and $(1-\alpha)D(i-1)P_p\{i\}$ in the peripheral part. The expected extra distance shall be the summation of these two terms across all $i \geq 2$.

These longitudinal, lateral and extra distances per bus round trip add up to the formulas for Q_c and Q_p , given that there are $2N/H$ buses in operation per hour.

¹⁸ The above sweeping strategy is used in the cost derivation because of (i) simplicity for formulation (in closed-form) and implementation in the real world, and (ii) conservativeness for comparison with other systems. In practice, other heuristic routing strategies (e.g., insertion heuristics) can be used for pick up and drop off passengers between two consecutive check points so as to further improve the system performance..

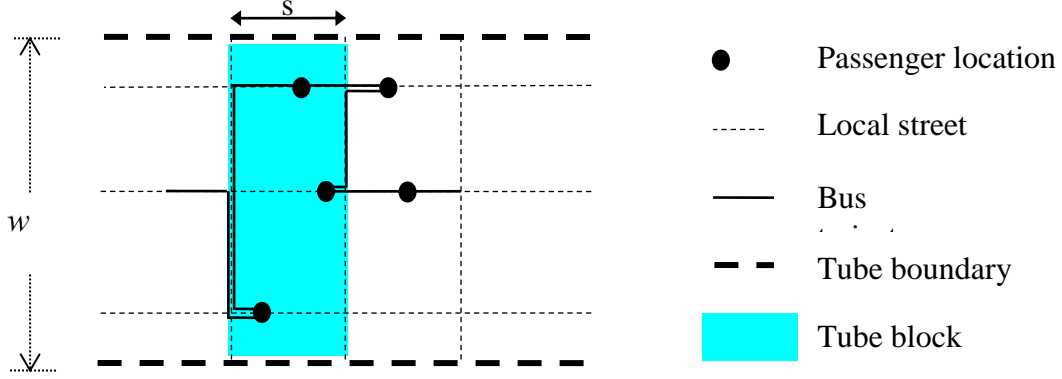


Figure A.2. The extra longitudinal distance for additional passengers.

Result 3: The expected waiting time per passenger is given by $w = \frac{H}{2} \left[\frac{N-1}{N} \left(1 - \frac{\alpha^4}{N} \right) + \frac{(1-\alpha^2)^2}{2} + 1 \right]$

Proof: Note that we have assumed that (i) the passengers randomly choose the transfer location in case they have more than one option, and (ii) headway control strategies are implemented to eliminate irregularities in bus arrival headways at the check points. The waiting time is obtained by multiplying the waiting time per transfer by the number of transfers, and adding it to the waiting time at the origin. Each waiting time (at the transfer or the origin) is approximately $\frac{H}{2}$, and the number of transfers is given by Result 1.

Result 4: The average bus speed satisfies $\frac{1}{v_c} = \frac{1}{v} + \frac{2\tau\lambda D^2 H/N}{QH/2N}$.

Proof: Without losing generality, we assume that during each stop we pick up or drop off exactly one passenger. From Result 0, during each trip, the number of passengers in the central

square is $\frac{\lambda}{2} a_{inner} H = \frac{\lambda}{2} \frac{D^2 \alpha^2}{N} H$ and the number of passengers in the peripheral quadrants is

$\lambda a_{outer} H = \lambda \frac{D^2 (1 - \alpha^2)}{2N} H$. Therefore the total number of stops is $2 \left(\lambda \frac{D^2 \alpha^2}{2N} H + \lambda \frac{D^2 (1 - \alpha^2)}{2N} H \right) =$

$\frac{\lambda D^2 H}{N}$. The bus needs to overcome time over distance $\frac{1}{v}$, and stop for τ time per passenger

during a round trip with distance $QH/2N$. Hence, $\frac{(QH/2N)}{v_c} = \frac{(QH/2N)}{v} + 2(\tau) \lambda D^2 H / N$, which leads

to the result.

Result 5: The expected in-vehicle travel distance and travel time per passenger are

$E = \rho_p E(R_p) + \rho_c E(R_c)$ and $T = \frac{E}{v_c}$ respectively, where

$$E(R_p) \approx \frac{2 - 3\alpha + \alpha^3}{3} D \text{ and } E(R_c) = \left(\frac{11}{12} \alpha - \frac{1}{12} \alpha^3 - \frac{1}{12} \alpha^5 \right) D$$

are the expected in-vehicle longitudinal distances per passenger trip in the peripheral part and the central square, respectively, and ρ_c and ρ_p are the ratios of the expected total distance over the expected longitudinal distance for the central and peripheral parts, respectively.

Proof: Figure A3 shows a quarter of the entire square region. The longitudinal distance between a random location (x, y) in the northern quadrant and the northern boundary of the central square

is $\left(y - \frac{d}{2} \right)$. Based on symmetry we conduct integral on $\frac{1}{8}$ of the peripheral region to calculate the

expected longitudinal distance per passenger in the peripheral quadrants. While doing so we

consider various combinations of trip origin/destination locations (i.e., in the central square or peripheral quadrants)¹⁹. It can be verified that

$$E(R_p) \approx 2 \frac{8}{D^2} \int_{\frac{d}{2}}^{\frac{D}{2}} \int_0^y (y - \frac{d}{2}) dx dy = 2 \frac{2D^3 - 3D^2d + d^3}{6D^2} = \frac{2 - 3\alpha + \alpha^3}{3} D.$$

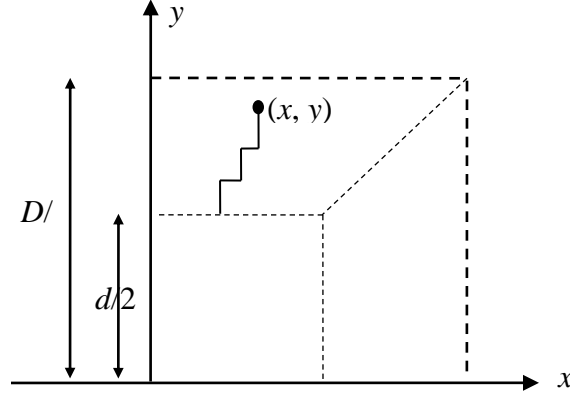


Figure A.3. The lateral distance for pick-up or drop-off passengers.

To compute $E(R_c)$, we again consider cases 1-3 regarding relative locations of the trip origin and destination (see the proof for Result 1). The expected longitudinal distances in these cases are $\frac{2d}{3}$, $\frac{5d}{6}$, and $\frac{11d}{12}$, respectively, and the corresponding probabilities are α^4 , $2\alpha^2(1-\alpha^2)$, and $(1-\alpha^2)^2$. Then,

¹⁹ While calculating the expected longitudinal distance in the peripheral area, we have assumed that all trips, if only starting and ending both in the peripheral part, shall travel through the central square. In fact, there is a very small probability, $\frac{1}{4N}(1-\alpha^2)^2$, that the origin and destination are both in the same peripheral part of the same tube, such that the trip will not go through the central square. The exact formula for $E(R_p)$, when considering this possibility,

would have been $\left[1 - \frac{1-\alpha^2}{4N}\right] \frac{2-3\alpha+\alpha^3}{3} D + \left[\frac{(1-\alpha^2)^2}{4N}\right] \frac{2(1+2\alpha-2\alpha^2-\alpha^3)}{15(1+\alpha)^2} D$. It can be shown, however, that

for most realistic examples, ignoring such a possibility would make no practical difference to the estimated value of E . In addition, it is conservative for us to ignore such a possibility, because in doing so we are (slightly) over-estimating E , and hence in favor of other transit systems.

$$E(R_c) = \alpha^4 \frac{2}{3}d + 2\alpha^2(1-\alpha^2) \frac{5}{6}d + (1-\alpha^2)^2 \frac{11}{12}d = \left(\frac{11}{12}\alpha - \frac{1}{12}\alpha^3 - \frac{1}{12}\alpha^5 \right) D.$$

To compute the lateral movement of each bus, we define ρ_c and ρ_p as the ratio of the total distance over the longitudinal distance. In the central square the longitudinal distance during one bus round trip is $2\alpha D$ and the total distance is $Q_c H / 2N$ (as shown in Result 2), and then

$$\rho_c = \frac{Q_c H / 2N}{2\alpha D}.$$

In the periphery the longitudinal distance during one bus round trip is $2D(1-\alpha)$ and the total distance is $Q_p H / 2N$, then

$$\rho_p = \frac{Q_p H / 2N}{3D(1-\alpha)}.$$

The analysis above yields the formula for E after simple algebraic manipulations. Then, the total expected travel time for each passenger trip is obviously

$$T = \frac{E}{v_c} = \frac{\rho_p E(R_p) + \rho_c E(R_c)}{v_c}.$$

Result 6: The vehicle occupancy is given by $O = \frac{\lambda H D^2}{N} \max\left\{ \frac{1-\alpha^2}{2\alpha}; \frac{3+2\alpha^2-3\alpha^4}{8\alpha} + \frac{D}{w} \frac{(1-\alpha^2)^2}{32} \right\}.$

Proof: This formula is essentially the same as Result 8 in Daganzo (2010).

The above analytical results for all different cost components have been validated by discrete-event simulations under the same assumptions.

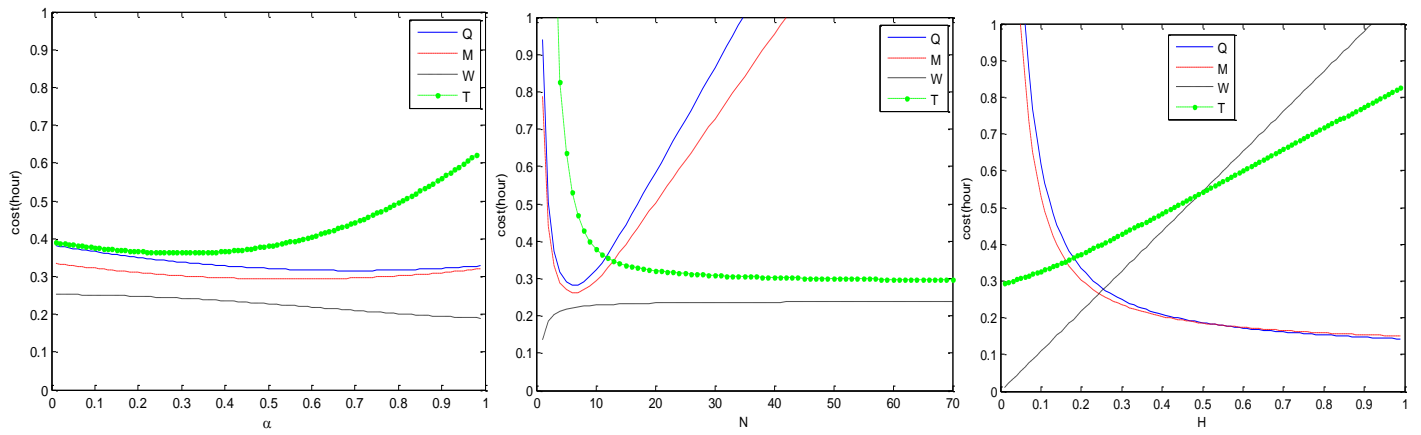
Appendix B

In this appendix we illustrate how decision variables α , N , and H influence the cost components Q , M , W , and T according to (1)-(12). We use $D=10 \text{ km}^2$ and $\lambda=10$ passengers per km^2 per hour.

As shown in Figure B1(a), the agency costs (Q and M) and the passenger waiting time W all decrease only slightly with α ; however the in-vehicle travel time T is high for both small α (i.e., a grid network) and large α (i.e., a hub-and-spoke network). As such, the optimal value of α is likely to be somewhere in the middle between 0 and 1, implying that a hybrid combination of grid and hub-and-spoke network will be desirable.

Figure B1(b) shows that the agency costs are high for either small or large N . When N is small, the tubes are wide, and the high agency cost is probably due to a large amount of lateral movements per passenger. On the other hand, when N is large, the tubes are narrow, and the agency cost could still be high due to the large number of buses needed in the system. The passengers' waiting time almost remains constant across all N , but the in-vehicle travel time decreases dramatically until N increases to a moderate number. This sharp decrease is probably because of the significant reduction in cumulative lateral travel distances that a passenger experiences. This curve flattens out when N continues to increase, implying that as long as the tube width is not too wide, the lateral travel would not significantly increase the system cost.

When headway H increases, the agency costs Q and M both decrease to a nonzero value; however the waiting time and the in-vehicle travel time increase almost linearly. These trends are shown in Figure B1(c).



(a) $N=10$ and $H=0.21$

(b) $\alpha=0.50$ and $H=0.21$

(c) $\alpha=0.50$ and $N=10$

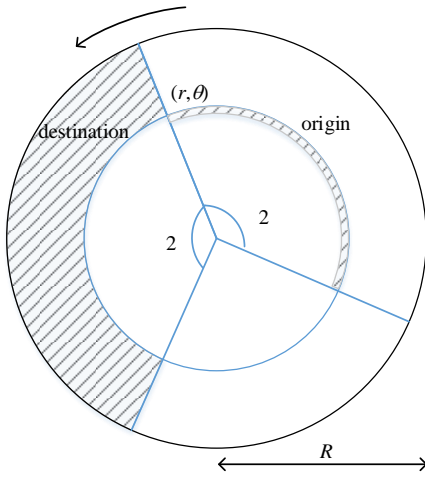
Figure B.1. Sensitivity of Q , M , W and T to α , N , and H .

Appendix C

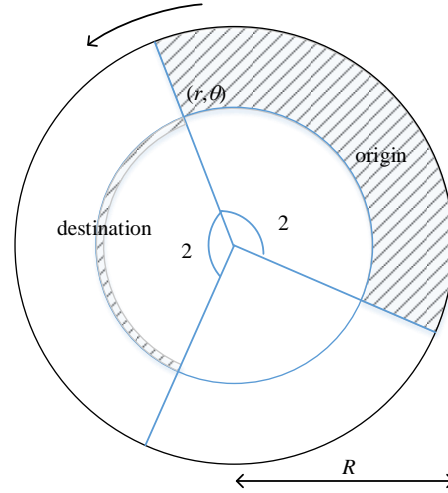
The flux in counterclockwise direction can be obtained by this equation:

$$F_{CC}(r, \theta) = \int_{\theta_1=\theta-2}^{\theta} \left[\int_{r_2=r}^R \int_{\theta_2=\theta_1}^{\theta_1+2} \delta(r, \theta_1, r_2, \theta_2) r_2 d\theta_2 dr_2 \right] d\theta_1 + \int_{\theta_2=\theta}^{\theta+2} \left[\int_{r_1=r}^R \int_{\theta_1=\theta_2-2}^{\theta_2} \delta(r_1, \theta_1, r, \theta_2) r d\theta_1 dr_1 \right] d\theta_2 .$$

The first term shows the passengers who were picked up in the circular slice with $\tilde{r}=r$ and $\theta-2 \leq \tilde{\theta} < \theta$ and their destination is in the hatched area with $r \leq \tilde{r} \leq R$ and $\theta < \tilde{\theta} < \theta+2$ (Figure C.1.a). The second term represents the passengers who were picked up at the hatched area with $r \leq \tilde{r} \leq R$ and $\theta-2 < \tilde{\theta} < \theta$ and their destination is in the circular slice with $\tilde{r}=r$ and $\theta < \tilde{\theta} < \theta+2$ (Figure C.1.b).



(a)



(b)

Figure C.1. Flux for counterclockwise circular direction buses.

Clockwise direction flux can be obtained using this equation:

$$F_{CW}(r, \theta) = \int_{\theta_1=\theta}^{\theta+2} \left[\int_{r_2=r}^R \int_{\theta_2=\theta_1-2}^{\theta_1} \delta(r, \theta_1, r_2, \theta_2) r_2 d\theta_2 dr_2 \right] d\theta_1 + \int_{\theta_2=\theta-2}^{\theta} \left[\int_{r_1=r}^R \int_{\theta_1=\theta_2}^{\theta_2+2} \delta(r_1, \theta_1, r, \theta_2) r d\theta_1 dr_1 \right] d\theta_2 .$$

The first term

indicates the passengers who were picked up in the circular slice with $\tilde{r}=r$ and $\theta < \tilde{\theta} < \theta+2$ and their destination is in the hatched area with $r \leq \tilde{r} \leq R$ and $\theta-2 < \tilde{\theta} < \theta$ (Figure C.2.a). The second term represents the passengers who were picked up in the hatched area with $r \leq \tilde{r} \leq R$ and $\theta < \tilde{\theta} < \theta+2$ and their destination is in the circular slice with $\tilde{r}=r$ and $\theta-2 \leq \tilde{\theta} < \theta$ (Figure C.2.b).

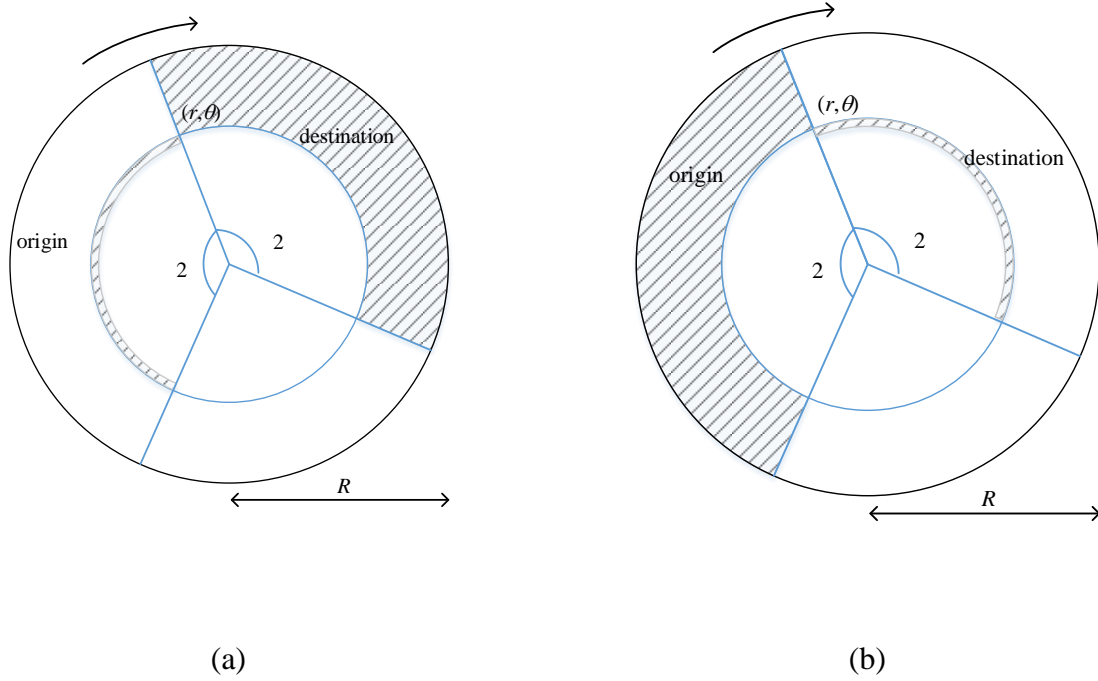


Figure C.2. Flux for clockwise circular direction buses.

$$F_I(r, \theta) = \int_{r_1=r}^R \left[\int_{r_2=0}^R \int_{\theta_2=\theta+2}^{\theta-2+2\pi} \delta(r_1, \theta, r_2, \theta_2) r_2 d\theta_2 dr_2 \right] r dr_1 + \int_{r_1=r}^R \left[\int_{r_2=0}^r \int_{\theta_2=\theta-2}^{\theta+2} \delta(r_1, \theta, r_2, \theta_2) r_2 d\theta_2 dr_2 \right] r dr_1 \quad \text{calculates the}$$

radial inner direction flux. In this equation, the first term denotes the passengers who were picked up in the radial slice with $\tilde{\theta} = \theta$ and $r \leq \tilde{r} \leq R$ and their destination is in the hatched area with $0 \leq \tilde{r} \leq R$ and $\theta + 2 < \tilde{\theta} < \theta + 2\pi - 2$ (Figure C.3.a). The second term indicates the passengers who were picked up in the radial slice area with $\tilde{\theta} = \theta$ and $r \leq \tilde{r} \leq R$ in and their destination is in the circular slice with $0 \leq \tilde{r} \leq r$ and $\theta - 2 \leq \tilde{\theta} < \theta + 2$ (Figure C.3.b).

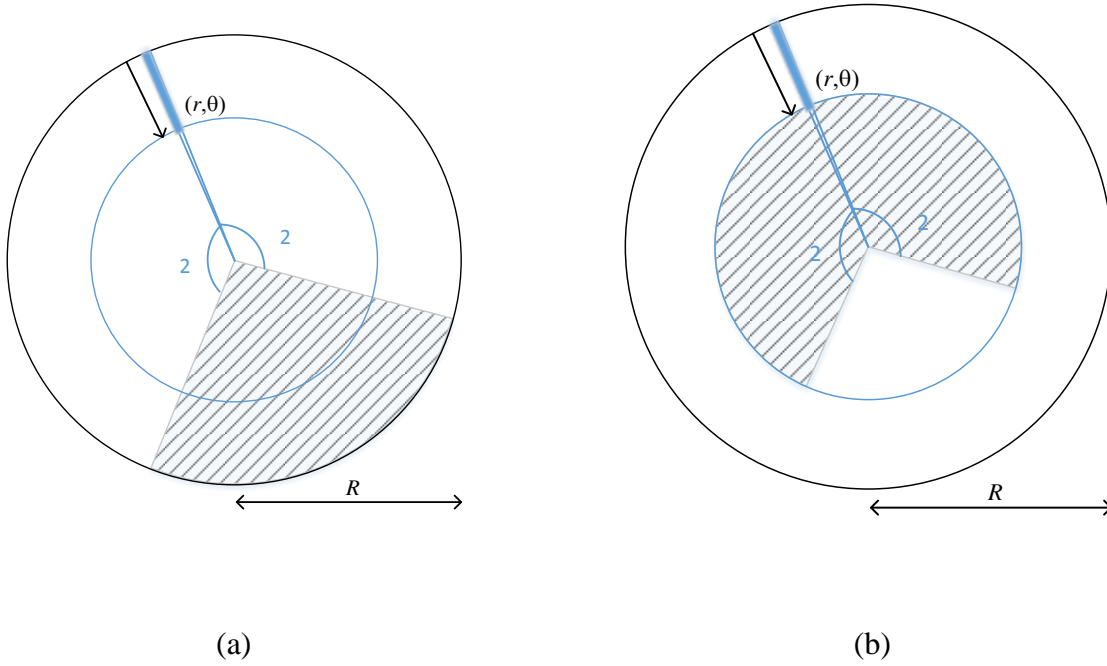
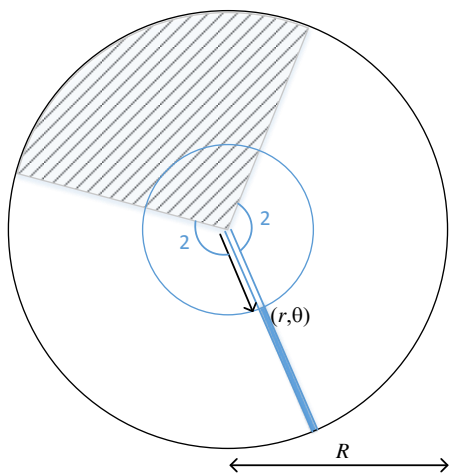


Figure C.3. Flux for radial inner direction buses.

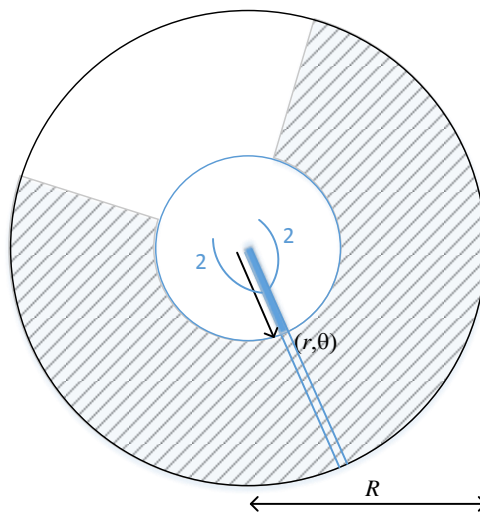
And at the end, this equation,

$$F_O(r, \theta) = \int_{r_2=r}^R \left[\int_{\eta_1=0}^R \int_{\theta_1=\theta+2}^{\theta-2+2\pi} \delta(r_1, \theta_1, r_2, \theta) r d\theta_1 dr_1 \right] r_2 dr_2 + \int_{r_2=0}^r \left[\int_{\eta_1=r}^R \int_{\theta_1=\theta-2}^{\theta+2} \delta(r_1, \theta_1, r_2, \theta) r d\theta_1 dr_1 \right] r_2 dr_2, \quad \text{computes the}$$

radial outer direction flux. In this equation, the first term denotes the passengers with origins in the hatched area with $0 \leq \tilde{r} \leq R$ and $\theta+2 < \tilde{\theta} < \theta+2\pi-2$ and their destination is in radial slice with $\tilde{\theta}=\theta$ and $r \leq \tilde{r} \leq R$ (Figure C.4.a). The second term indicates the passengers who were picked up in the radial slice area with $\tilde{\theta}=\theta$ and $0 \leq \tilde{r} \leq r$ and their destination is in the circular slice with $r \leq \tilde{r} \leq R$ and $\theta-2 \leq \tilde{\theta} < \theta+2$ (Figure C.4.b).



(a)



(b)

Figure C.4. Flux for radial outer direction buses.